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Proefschrift ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, prof. dr. R.A. de Moor, in het openbaar te verdedigen ten overstaan van een door het college van dekanen aangewezen commissie in de aula van de Universiteit op vrijdag 2 november 1990 te 14.15 uur door

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PREFACE

Part of the research reported in this thesis was financed by The Netherlands Organization for the Advancement of Pure Research (ZWO). This part was carried out when I worked at Tilburg University (from December 1985 until August 1988). The other part was carried out at Groningen University, at which I started to work in September 1988.

Data were provided by The Netherlands Organization for Strategic Labor Market Research (OSA), The Netherlands Central Bureau of Statistics (CBS) and by Geert Ridder.

Chapter 2 resembles van den Berg (1990c), which is published in the Economic Journal.

Chapter 3 is virtually identical to van den Berg (1990b), which appeared in the Review of Economic Studies.

Chapters 4 and 5 are based on van den Berg (1989) and van den Berg (1990a), respectively, which are under revision for publication.

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I am grateful to a number of colleagues who made helpful comments on papers underlying this thesis. In particular I would like to mention Peter Kooreman, John Rust, Andrew Chesher, Tony Lancaster, Wiji Narendranathan, Ken Burdett, Maarten Lindeboom and Richard Blundell. Further, I thank those colleagues at Tilburg University and Groningen University who helped creating a good and professional atmosphere at the Department. The results in Chapter 4 benefited from the computational assistance of Rob Aalbers. Jacobien Bruining accurately typed versions of Chapters 2, 3 and 4. Finally, I thank Els Boerrigter for a lot of reasons that I can't spell out in a few lines.

Groningen, August 1990.

It is impossible to achieve the aim without suffering.

J.G. Bennett

CONTENTS

CHAPTER 1

INTRODUCTION

The four essays in this dissertation deal with microeconometric models of individual labour market behaviour. In particular, the models try to explain transitions between different states on the labour market and durations in those states. Job search theory serves as the underlying framework of the models. This theory explicitly incorporates various forms of uncertainty. It is dynamic in the sense that it concentrates on behaviour over time. Job search theory has a distinctly neoclassical flavour, since it assumes that decisions of individuals are governed by the aim to maximize their own expected utility.

The empirical analysis in this dissertation is structural. This means that it is attempted to uncover the primitive objects underlying the strategies of the individuals. In this case these objects include the individuals' preferences and the probability distributions characterizing the uncertainty about future events. Structural empirical analysis enables one to make detailed inferences on the determinants of job and unemployment durations and on the characteristics of job search. For instance, a distinction can be made between choice and chance components of the transition rate from unemployment to employment. Furthermore, it is possible to examine the reasons for the (in-)effectiveness of policies aimed at, for instance, a reduction of unemployment durations or an increase of job mobility. Generally, the models considered are highly nonlinear in the parameters and contain latent (i.e. unobserved) variables. The estimation methods used are maximum likelihood and, occasionally, nonlinear least squares. The four studies, Chapters 2 through 5 in this thesis, all start with an extensive introduction of their own, so this introductory chapter is kept brief. In what follows we briefly point at the distinctive features of the different studies.

Up to now a number of empirical studies on unemployment using structural search models have been published. To our knowledge, no previous study has estimated a model which allows for transitions from the state of unemployment into the state of nonparticipation. However, by now there is ample evidence that a large portion of the flow out of unemployment consists of transitions into nonparticipation. In Chapter 2 we construct and estimate a structural job search model that allows for such transitions. For unemployed individuals the optimal strategy with regard to job offers is affected by the rate at which transitions into nonparticipation take place. As a result, characteristics of the search process, like the probability of accepting a job offer, depend on this transition rate.

Another objection to the structural empirical analyses in the literature on unemployment is that the job search models used so far do not take into account that wage increases during employment may be expected. The optimal strategy of unemployed individuals is likely to be dependent on wage changes that may occur after the acceptance of a job. In Chapter 2 we therefore also estimate an extended version of the model, which deals with these aspects. For both model versions we examine the effect of a decrease in unemployment benefits on unemployment duration and we explain the magnitude of this effect in terms of the characteristics of the search process.

Chapter 3 extends the literature on job search theory by weakening the so-called stationarity assumption. Generally, the search models used in structural empirical (and theoretical) studies on unemployment are stationary. This implies that variables like unemployment benefits or the rate of arrival of job offers are assumed to be constant over the spell of unemployment, which is often at variance with reality. Moreover, various reduced-form empirical studies on unemployment duration indicate a significant duration dependence of the transition rate from unemployment into employment, which is generally interpreted as evidence against stationarity. In Chapter 3 we examine the consequences of nonstationarity in continuous-time job search models in which the explanatory variables (the level of unemployment benefits, the job offer arrival rate and the wage offer distribution) are allowed to vary over time in a very general way. Throughout the chapter we are concerned with job searchers with perfect foresight in the sense that they are assumed to correctly anticipate changes in the values of the explanatory variables. A general differential equation describing the evolution of the optimal strategy over time is derived. The more specific the assumptions made on the time paths of the explanatory variables, the more detailed our inferences on the solution of this differential equation. We also present comparative dynamics results concerning the shift in the time path of the optimal strategy if some particular time path of an explanatory variable is replaced by another.

In addition to these theoretical results, Chapter 3 contains an empirical illustration of the importance of allowing for nonstationarity. In The Netherlands, the benefits level during the first few years of unemployment is related to the pre-unemployment wage while the level after those years is determined by the public assistance system. As a consequence, benefits generally decrease substantially when the duration of unemployment exceeds a certain period. We estimate a nonstationary structural model that allows for this. Given the parameter estimates of the model, we can analyze the effects of the decrease of the benefits level on the time path of the optimal strategy and the exit rate out of unemployment. Also, the estimated model can be used to examine the effects of changing (part of) the existing time path of the level of benefits.

Chapter 4 provides a structural empirical analysis of the labour-market behaviour of employed individuals, using a so-called on-the-job search model. Search theory is a popular tool for explaining job mobility; by now there is an extensive theoretical literature on search on the job for better jobs. However, to our knowledge there are no published papers that empirically test the on-the-job search model. We construct and estimate a model that pays particular attention to the costs associated with moving to another job, since it is likely that these are among the most important of the factors causing inflexibility of the labour market. It is shown that under certain conditions the optimal strategy of an employed individual can be characterized by a reservation wage. The parameter estimates are used to examine the effects of changes in the level of the costs of moving to another job and the value of the job offer arrival rate on the optimal strategy and the duration of a job.

Chapter 5 is of a theoretical nature and does not contain empirical results. The issue of this chapter is the relationship in job search models between, on the one hand, the rate at which an unemployed individual obtains job offers and, on the other hand, the expected duration of unemployment. Generally it is acknowledged that an increase of the job offer arrival rate has two opposite effects on the expected duration of unemployment. First, there is a negative effect because of the increased expected number of occasions on which one is able to leave unemployment. Secondly, there is a positive effect because of the increased selectivity of the searcher in face of this increased opportunity to leave unemployment. In the literature sufficient conditions on the shape of the wage offer distribution have been derived that ensure that an increase of the arrival rate causes a decrease of the expected unemployment duration. However, these conditions are not satisfied for the families of distributions that can realistically represent wage offer (and income) distributions. Therefore they seem to be of limited practical interest in guiding the interpretation of estimates of reduced-form models of unemployment durations. Also, since the assumed families of wage offer distributions in structural empirical analysis generally do not satisfy those conditions, the suspicion may arise that the estimates of structural job search models are sensitive with respect to the assumed family. In Chapter 5 it is shown that the previously derived sufficient conditions can be weakened at no cost, to include virtually every conceivable (wage offer) distribution. Thus it seems that the scope for the problems in empirical analysis which were mentioned above is narrowed a great deal.

Chapter 6 contains a brief summary and evaluation of the various results.

CHAPTER 2

SEARCH BEHAVIOUR, TRANSITIONS TO NONPARTICIPATION AND THE DURATION OF UNEMPLOYMENT

2.1. Introduction

In this chapter we examine the estimation of a structural job search model using data on individual unemployment durations. The model allows for transitions from unemployment to nonparticipation. In an extended version of the model we deal with the influence of on–the–job search and prospective wage increases on search behaviour of the unemployed.

In empirical studies on unemployment duration the reduced–form approach, in which only hazards of the duration distribution are estimated (see for example Lancaster (1979)), seems to be replaced gradually by a structural approach. The latter way of modelling is characterized by the explicit use of the framework of job search theory in empirical analysis. The results from such analyses can be used for inferences about the behaviour of the unemployed. In particular a distinction can be made between choice and chance components of the transition rate into employment.

Several empirical studies using structural search models have been published (see for example Yoon (1981), Lancaster & Chesher (1983), Lynch (1983), Narendranathan & Nickell (1985), Ridder & Gorter (1986), Wolpin (1987)), some of which use a very restricted model specification (notably the first three references). None of those studies uses a model that allows for transitions from unemployment to nonparticipation. In reality an individual who is unemployed and actively searching for a job may drop out of the labour force at some point of time during unemployment. It may be that the papers referred to do not take account of transitions into nonparticipation because the data used are not rich enough to make the distinction between the states of unemployment and nonparticipation. This can be the case if the data collection is based on the receipt of unemployment benefits. Another cause for not taking account of such transitions may be that in the time those data were collected (typically the seventies) the occurrence of such transitions was less prominent. However, by now there is much evidence that a large portion of the flow out of unemployment consists of transitions into nonparticipation (for a survey of the literature, see Micklewright (1988) who also forcefully argues that the state of nonparticipation should be incorporated in duration models of the labour market, especially if one is interested in the effects of benefits on unemployment duration). In the sample we use, almost 30% of all spells of unemployment ends up in a transition into nonparticipation. Therefore we estimate a structural job search model that allows for such transitions.

Further, up to now the structural models used in empirical analyses do not take into account that wage increases during employment may be expected. Wages can increase for several reasons such as accumulation of human capital or transitions from jobs with lower wages to jobs with higher wages without intervening spells of unemployment (on the job search, see for example Mortensen (1986)). The optimal strategy of an unemployed individual is likely to be dependent on changes of wages and jobs that occur after the acceptance of a job. We estimate an extended version of the model, which deals with these aspects.

In Section 2.2 we discuss the specification of the model. As a starting point we take a search model that resembles Narendranathan & Nickell's (1985) model. This model is extended to allow for transitions into nonparticipation. We outline how the model may be given an alternative interpretation which is more realistic with regard to the process of search. This interpretation allows for knowledge of the wage rate associated with a vacancy before one responds to that vacancy, i.e. before the job is actually offered. Section 2.3 contains a description of the data and a discussion of the empirical implementation of the model. The structural model is estimated by ML using the Newton–Raphson algorithm. The estimation method is analogous to that used by Narendranathan & Nickell (1985) and Wolpin (1987) in the sense that for every individual in the sample, for every iteration, the optimal search strategy, which follows from a dynamic programming problem, needs to be solved. Section 2.4 deals with the estimation of the wage offer distribution. Section 2.5 presents the main results. We present estimates of the job offer arrival rate, the transition rate into nonparticipation and the utility function. For distinct age categories and levels of education we present sample averages of the main characteristics of the job search process. From a policy viewpoint it may be of interest to see whether a decrease in unemployment benefits has any influence on duration. If not, this may lead to a re-evaluation of benefits as a policy tool. Therefore we give special attention to the effects of changes in benefits on the reservation wage and the expected duration. In Section 2.6 the construction of the extended model is described and the results of the estimation of the extended model are discussed. Section 2.7 concludes.

2.2. The model

2.2.1. *Job search theory and model specification*

We start by presenting the basic job search model for unemployed individuals who are searching sequentially for jobs until a suitable one has been found (for surveys on job search theory, see Mortensen (1986) or McKenna (1985)). Job offers arrive randomly in time at the arrival rate λ. Such job offers are random drawings from a wage offer distribution $F(w)$. During unemployment a benefit b is received. The variables λ, b and w are measured per unit time period. Unemployed individuals aim at maximization of their expected discounted lifetime utility (over an infinite horizon). To begin with we also assume that once a job is accepted it will be held forever at the same wage.

The per-period utility function is a separable function of two arguments, income and state:

$$\text{utility (income} = x, \text{ state} = \text{employment)} \qquad = v^*.u(x)$$

$$\text{utility (income} = x, \text{ state} = \text{unemployment)} \qquad = v.u(x)$$

in which v^* and v do not depend on income x. This utility function was first used by Nickell (see Lancaster & Chesher (1983)). The function u is increasing in its argument and may take account of risk aversion. Somewhat loosely we call v the disutility of unemployment. Note that v and v^* may differ not only because of the difference between the amount of leisure in both states, but also because of other differences between those states, like the difference in social status. We normalize by setting $v^* = 1$.

In the sequel only stationary job search models are considered. This means that we take λ, b, u, v and $F(w)$ to be independent of unemployment duration and calendar time and independent of all events during unemployment. Obviously this is not very realistic. The level of unemployment benefits depends generally on the elapsed duration of unemployment. The job offer arrival rate may decrease during unemployment as a result of the stigma that the long-term unemployed may have. Further, λ, b and $F(w)$ may change due to business cycle effects. The motivation for adopting the stationarity assumption is basically the same as it was in the other empirical studies using structural search models (see for example Lancaster & Chesher (1983) and Narendranathan & Nickell (1985)). That is, when estimating a nonstationary model the computational difficulties are likely to be even more burdensome, so it seems a good strategy to start off with a stationary model. (For an analysis of nonstationarity in job search theory, see Chapter 3.) In Section 2.5 we turn to the effects that the presence of nonstationarity might have on the estimation results.

The optimal strategy of an unemployed individual in the model sketched above can be characterized by a fixed reservation wage ϕ. A job offer is accepted if its wage exceeds ϕ while a wage that is smaller than ϕ induces one to reject the offer and search for a better one. The transition rate from unemployment into employment θ can be written as the product of the job offer arrival rate and the conditional probability of accepting a job offer.

(1) $\qquad \theta = \lambda\overline{F}(\phi) \qquad\qquad \overline{F} = 1-F$

In reality an individual who is unemployed and actively searching for a job may drop out of the labour force, at some point in time during unemployment. This may be the result of a personal decision such as deciding to dedicate all available time to household activities. It can also be a forced transition, for example when he is conscripted or when he becomes disabled or when he retires. All these cases can be labelled as transitions out of unemployment into nonparticipation.

Flinn & Heckman (1982) present a three-state structural search model which could serve as a starting point for our model. In this three-state model the distribution of returns of nonparticipants enters the equations that describe the behaviour of the unemployed. This implies that data on returns of nonparticipants are needed in order to estimate the model. Such data are not available. Therefore we adopt a reduced-form model of the transitions from unemployment into nonparticipation. Specifically, such transitions are assumed to occur according to a Poisson process with a parameterized transition rate ζ.

The optimal strategy of an unemployed individual depends on the expected utility of becoming a nonparticipant. If the latter is high with respect to the expected utility of becoming employed then it is optimal to accept a job offer only if the wage corresponding to it is very high. Let x denote the flow of income of a nonparticipant. We make the assumption

(2) $\qquad Eu(x) = u(b)$

For a lot of cases the income flow after becoming a nonparticipant is close to the benefit level (for example when an unemployed individual becomes disabled, when he retires, when he is conscripted, when he returns to school and applies for social assistance). If the dispersion of the distribution of x is small, which we expect to be the case, then $Ex \approx b$ implies that $Eu(x) \approx u(b)$. To sum up, we do not assume anything about the distribution of the income flow x in the state of nonparticipation except that equation (2) holds. In addition, we assume that the state of nonparticipation is absorbing and, for the moment, we assume that the non–pecuniary component v of per–period utility in nonparticipation is the same as that in unemployment.

As an additional condition for stationarity to hold we require that ζ is constant (though possibly different across individuals). Again this may not be very realistic. Individuals may enter nonparticipation at an increasing rate when they become discouraged about their chances on the labour market. This in turn may happen more frequently among the long–term unemployed. However, the empirical relevance of this effect is not well known. Reduced–form studies in which ζ and other transition rates are estimated either do not have enough observed transitions from unemployment into nonparticipation in the data to estimate a duration dependent ζ (e.g. Ridder (1987)), or stick to the assumption that ζ is constant (e.g. Burdett, Kiefer, Mortensen & Neumann (1984) and Blau & Robins (1986a)).

In the appendix to this chapter it is proved that the reservation wage ϕ, which characterizes the optimal strategy in the model, satisfies the following equation, in which ρ denotes the subjective rate of discount

$$(3) \qquad u(\phi) = v.u(b) + \frac{\lambda}{\rho+\zeta} \cdot \int_{\phi}^{\infty} (u(w)-u(\phi))dF(w)$$

The exit rate out of unemployment is equal to the sum of θ and ζ, with θ given by equation (1). Because θ and ζ do not depend on duration or on time or on events during unemployment this implies that the unemployment duration has an exponential distribution with parameter $\theta + \zeta$.

2.2.2. *An alternative interpretation*

It can be argued that the modelling of the search process so far is not very realistic. Generally one knows the wage rate associated with a vacancy before one responds to that vacancy, i.e. before the job is actually offered. Narendranathan & Nickell (1985) constructed a search model that deals with this. Job vacancies arrive according to a Poisson process with arrival rate q_1. A vacancy is characterized by a random drawing from a distribution of wages associated with the flow of vacancies, G(w). The decision whether to apply or not is made with knowledge of the wage corresponding to the vacancy. If one does apply, then there is a (known) probability of $q_2(w)$ that the job will actually be offered. The dependence of q_2 on w represents increased competition for vacancies with higher wages.

It is straightforward to show that the model developed in Subsection 2.2.1 is equivalent to the model described here. To see this, set

$$(4) \qquad \lambda = q_1 \cdot \int_0^\infty q_2(\omega) dG(\omega)$$

$$(5) \qquad F(w) = \frac{\int_0^w q_2(\omega) dG(\omega)}{\int_0^\infty q_2(\omega) dG(\omega)}$$

Consequently, the estimation results of the original model can be reinterpreted according to equations (4) and (5). Narendranathan & Nickell (1985) make the convenient assumption that

$$(6) \qquad q_2(w) = q_3(w) . q_4$$

in which q_3 depends on w only, while q_4 represents the dependence of q_2 on personal characteristics. If (6) holds then F(w) in (5) does not depend on q_4, i.e. does not depend on personal characteristics which influence the probability that the job is offered given application.

2.3. The data

2.3.1. *The data set*

The data set used is constructed from the Netherlands Socio-Economic Panel, a survey conducted by the Netherlands Central Bureau of Statistics. Since April 1984 a random sample of about 12000 individuals has been interviewed twice a year (in April and October). At every interview except the first one, respondents were asked to recall their labour market history for the past 6 months, that is, they were asked between which dates in the last 6 months they had a job, between which dates they were unemployed and searching for a job and between which dates they were doing something else. The latter category of activities includes being disabled, doing unpaid work in the household, being retired (the retirement age varies between (roughly) 55 and 70 years and centres on 65 years), being in full-time training, being conscripted and just doing nothing. At the first interview the observation period is extended to 12 months. Given present information we have labour market histories for 2.5 years, from May 1983 up to October 1985.

For our purposes we selected 223 men aged between 17 and 65, who reported that at the moment of the first interview (April 1984) their main activity was being unemployed and searching for work. We determined for how long they were unemployed and searching for work at that moment, and (using subsequent waves) also for how long they would remain unemployed and searching for work after that moment. By analogy with the renewal theory literature we call these durations the backward and forward recurrence times, respectively. For 40 individuals we could not construct the forward recurrence time because they were not interviewed in subsequent waves. These are mainly young people leaving their parents' home. Note that this might create a selection problem since these people might leave because they found a job elsewhere. We return to this issue in Section 2.5.

Of the backward and forward recurrence times, 64% and 39% are censored in the sense that it is only known that the realized time exceeds a certain value. Part of the 39% is due to respondents who drop out of the panel before October 1985. Of all 112 uncensored forward recurrence times 71% ended in a transition into employment. The other 29% became nonparticipants. This means that according to the labour market history as defined above there is a date such that the spell of unemployment ends on it while after that date the individual is doing something else than working in a paid job. Consequently, the state of nonparticipation covers the wide range of activities that was mentioned above as being included in the 'third' category of activities. The limited amount of observations in the sample prohibits a subdivision of the state of nonparticipation into different states.

By taking a closer look at the uncensored forward durations we observe a phenomenon that appears strange at first sight. Of the 112 uncensored forward recurrence times 54% seem to have ended at the day of an interview. That is, at wave n (n = 1,2,3) the individual reports that he is unemployed whereas at wave n+1 he reports that as of the date of the previous interview he has been in a different state. Clearly these people over-estimate the elapsed duration of the activities that they perform after leaving the state of unemployment. We have to account for these 'memory problems' when deriving the likelihood.

The data set provides a range of personal characteristics. We used the characteristics as reported in April 1984. Since we do not know the level of benefits that individuals obtained during spells of unemployment that started and finished between two successive waves of the panel, we decided to consider only those spells that contained the date of the first interview. Subsection 2.3.3, in which the explanatory variables in the model are discussed, contains a table with sample characteristics.

The data on income variables all count for the survey. The unemployment–insurance benefit variable is not imputed but instead is measured directly by asking respondents who are unemployed in the first wave of the panel what their net (after–tax) unemployment income was at the date of the interview. Benefits need not be constant throughout the spell. In The Netherlands in the beginning of the eighties the benefits level during the first years of unemployment is related to the pre–unemployment wage while after about two years it is determined by the public assistance system. However, if the benefits level related to the pre–unemployment wage is below the public assistance level, or if the individual did not have a job before becoming unemployed, then he obtains public assistance benefits from the beginning. Given the lack of information on pre–unemployment wages it is not possible to infer whether an individual in the sample faced decreasing benefits or not. In Subsection 2.5.2 we examine the consequences that ignoring decreases in benefits (if present in reality) have on the estimation results.

As said before the data do not contain information on the return from being a nonparticipant. In some cases nonparticipants can receive unemployment insurance benefits, for example if according to their own perception they do unpaid work in the household and do not search for a job while they are still officially registered as being unemployed. For most activities covered by the state of nonparticipation however the income level is not directly related to the unemployment insurance system. For example an individual who is retired or disabled obtains a fixed amount of money every month, usually supplemented by a pension if he is retired.

The data on the income variables that are used to estimate F(w) will be discussed in Section 2.4 as that section is devoted entirely to the estimation of F(w).

2.3.2. *Likelihood function*

In our stationary model the backward and forward recurrence time and the state of destination given exit from unemployment are stochastically independent (see for example Ridder (1984)). Because of this independence the individual log-likelihood contribution is simply the sum of three parts. The state of destination given exit from unemployment has a Bernoulli distribution with parameter $\theta/(\theta+\zeta)$. The forward recurrence time has an exponential distribution with parameter $\theta+\zeta$. By assuming that the individual entry rate into unemployment is constant before the moment of the first interview, the backward recurrence time follows this distribution as well. The forward and backward recurrence times are denoted as τ and t, respectively. The state of destination is denoted as ε with $\varepsilon = 1$ if the state is employment and $\varepsilon = 0$ if the state is nonparticipation. The occurrence of censoring and the occurrence of the so-called memory problems are taken to be exogenous. If τ is missing then this is taken to be exogenous as well.

First consider the state of destination. Let $c_1 = 1$ if τ is censored and $c_1 = 0$ otherwise. Let $c_2 = 1$ if τ is missing and $c_2 = 0$ otherwise. The part of the individual log-likelihood contribution L due to the state of destination is L_1,

$$(7) \qquad L_1 = (1-c_2)(1-c_1)(\varepsilon.\log \theta + (1-\varepsilon).\log \zeta - \log(\theta+\zeta))$$

So if τ is censored or missing then ε is not observed and consequently $L_1 = 0$.

Next consider the backward recurrence time. Let $c_3 = 1$ if t is censored and $c_3 = 0$ otherwise. The part of L due to t is L_2,

$$(8) \qquad L_2 = (1-c_3).\log(\theta+\zeta) - t.(\theta+\zeta)$$

If no memory problems are present then the part of L due to τ can be obtained by replacing in equation (8) $1-c_3$ by $(1-c_1)(1-c_2)$ and t by $(1-c_2)\tau$. Recall that memory problems are present if the data suggest that the spell of unemployment ended on the day at which the individual was being interviewed for the first, second or third time. For such individuals it can only be inferred that the spell ended somewhere between two subsequent interviews, say the n–th and the (n+1)th (n = 1,2 or 3). By assumption it is ruled out that transitions can be forgotten. One is inclined to think that when the spell of unemployment ends shortly before the (n+1)th interview the date of the transition will be reported more accurately than when the spell ends shortly after the n–th interview. This is confirmed by the fact that most reported transitions between two subsequent interviews took place less than three months before the second interview. Therefore, if a memory problem is present in the sense that a spell seems to have ended at the date of the first, second or third interview, then this is interpreted as evidence that the spell has ended between that date and three months later. Later on it will be examined whether the results are sensitive with respect to the assumption that memory problems can only occur if the transition takes place in the three month period after each interview. Let τ_1 denote the length of this three month period. Let $c_4 = 1$ if a memory problem is present and $c_4 = 0$ otherwise. The part of L due to τ is L_3,

$$(9) \qquad L_3 = (1-c_2).[\ (1-c_1).\{\ (1-c_4)(\log(\theta+\zeta) - \tau.(\theta+\zeta))$$

$$+ c_4.(\log(\ e^{-(\theta+\zeta)\tau} - e^{-(\theta+\zeta)(\tau+\tau_1)}\ ))\ \} + c_1.\{\ -\tau.(\theta+\zeta)\ \}\ ]$$

$$= (1-c_2).[\ -\tau.(\theta+\zeta) + (1-c_1)(1-c_4)\log(\theta+\zeta)$$

$$+ (1-c_1)c_4.\log(\ 1 - e^{-(\theta+\zeta)\tau_1}\ )\ ]$$

It is likely that similar to the occurrence of memory problems in the reported values of τ there may be problems in the reported values of t. In the sample almost no transitions into unemployment are reported for the first three months after April 1983. We assume that whenever a transition into unemployment occurred before July 1983, individuals with a memory problem report at the date of the first interview that they have been unemployed for more than a year. Consequently in case the reported censored t equals one year then this is interpreted as evidence that t exceeds nine months. Let t_1 denote the length of that nine month period. Equation (8) has to be modified to

$$(10) \qquad L_2 = (1-c_3)(\log(\theta+\zeta) - t.(\theta+\zeta)) - c_3.t_1.(\theta+\zeta)$$

The log-likelihood contribution L of an individual with known c_1, c_2, c_3, c_4, t, τ and ε is given by the sum of the right-hand sides of equations (7), (9) and (10). The structural parameters and functions of the job search model $(u,v,\rho,\lambda,F(w))$ enter the likelihood via θ (see equations (1) and (3)). The parameter ζ enters L both directly and indirectly via θ.

2.3.3. *The empirical implementation*

Now that we have specified the structural model and described the data we examine in this subsection the functional forms of the exogenous variables and discuss parameterizations. However, the wage offer distribution will be examined in Section 2.4 as that section is devoted entirely to the estimation of F(w).

The job offer arrival rate λ and the transition rate into nonparticipation ζ are written as exponential functions of observable exogenous variables x and z, respectively,

$$\lambda = \exp(x'\beta),$$

$$\zeta = \exp(z'\gamma)$$

The vector x includes variables which are of interest to employers, for example because they give an indication of the productivity of the job searcher. Examples are level of education (we distinguish between five levels: (1) no certificate after primary education, (2) lower secondary education, (3) secondary education, (4) higher vocational training, (5) university), age, nationality, whether the individual has had a job before (this was asked explicitly) and whether he is married. We include the local unemployment percentage as a (crude) indicator of labour market tightness. The vector x also includes a variable that depends on the number of working individuals in the household. If this number is high then the unemployed individual may have easier access to employers.

The vector z consists of variables which are important for the process of transiting into nonparticipation, either by chance or by choice. Obviously, age is important because young individuals may get drafted into the armed forces and older individuals retire or get disabled more often than younger ones. Furthermore, young unemployed individuals often return to school for additional training especially if they did not have any job before.

Note that some of the variables in x and z change, or may change, over time. It can be argued that, for the model to be correctly specified, this should explicitly be taken into account. However, by doing so, the analysis would be complicated enormously. Therefore we assume (in accordance with the bulk of the literature on both structural and reduced-form empirical duration analysis) that the rate at which the variables in x and z change and the magnitude of these changes are sufficiently small for the model in which these variables do not change to be reliably usable for inference.

Table 1 contains some sample characteristics of the explanatory variables in the model. From the previous subsection it is clear that sample averages of the duration variables are not informative. Therefore those are not presented.

Table 1. Sample characteristics.

variable	mean	standard deviation
benefits (guilders/week)	305	114
local % unemployment rate	18	2.7
age	32	11
level of education	2.0	1.0
# working in household	0.37	0.62
Dutch	0.91	
head of household	0.71	
married	0.48	
new entrant	0.11	

Similarly to Narendranathan & Nickell (1985) and Ridder & Gorter (1986) the utility function of income u is taken to be logarithmic. The subjective rate of discount ρ is fixed at 10% per year. In Section 2.5 we examine the robustness of the results with respect to changes in the functional form of u and with respect to the numerical value of ρ.

Non-wage income is not included in the model because figures on personal non-wage income components are not available in the first wave of the panel survey. A reduced-form estimation of θ with income of other household members included as a regressor in log θ showed that this variable has no influence at all on the transition from unemployment into employment. Therefore it was omitted in the structural model.

The estimation method we have employed was ML using the Newton-Raphson algorithm. Because of the assumptions that were made on the functional forms of F(w) (see Section 2.4) and u, it follows that equation (3) can be rewritten as an equation that can be solved numerically for ϕ with a high level of precision. Via equation (1) the likelihood contributions can then be calculated as a function of the parameters.

2.4. The wage offer distribution

The most natural way to obtain information on F(w) in a structural job search model is to use data on post-unemployment wages, for these are drawings from

F(w) truncated at ϕ. Combining such data with duration data makes it possible to estimate F(w) jointly with the other parameters in the model, provided that F(w) satisfies Flinn & Heckman's (1982) recoverability condition. However, as we saw in Subsection 2.3.1, in our sample there are only 79 transitions from unemployment into employment. Obviously we want to allow for different F(w) in different segments of the labour market. For some segments there are not enough post-unemployment wages available in order to be able to estimate F(w). For example there are only two individuals with a university degree who provide such wages. Therefore we take a totally different route in estimating F(w). We estimate F(w) a priori using data on individuals who were employed at the date of the first interview. Analogous to Narendranathan & Nickell (1985) the a priori estimation results serve to predict individual wage offer distributions for the unemployed. These predictions are plugged in when estimating the structural model.

Wages of employed individuals are not random drawings from F(w). A working individual accepted his present job because its wage exceeded his reservation wage when he was unemployed. Consequently, observed wages are drawn from a truncated distribution. However, the point of truncation (the reservation wage before obtaining the job) is unknown and cannot be estimated, because the level of unemployment benefits received before obtaining the current job is not available in the data set. In order to deal with this problem we use an ad hoc reduced-form wage model. The wage w is observed if and only if one is employed. Previous studies (for example Kiefer & Neumann (1979a)) assumed this to be equivalent to $w \geq \phi$, that is, w is observed if and only if it exceeds the reservation wage prior to employment. However, this is only true in a discrete time model in which exactly one job offer arrives per period (see Flinn & Heckman (1982)) which is a very strong assumption because it neglects various sources of the dynamics and uncertainty in the process of search. Therefore we take a latent variable y^* as determining whether one is employed: w is observed if and only if $y^* > 0$. The wage offer distribution F(w) is assumed to be log-normal with parameters μ and σ^2; $\mu = \eta' x_1$ with x_1 observed. The unobserved variable y^* is assumed to be a linear function of observed exogenous variables x_2 and an error term. Obviously every factor that influences F(w) influences y^* as well. Therefore the variables in x_1 are included in the set of variables in x_2. In order to allow for different values of the parameters of the wage model in different segments of the labour market the wage model is estimated separately for each segment. Details of the estimation and the results are given in van den Berg (1988). The wage offer distribution of an unemployed individual with characteristics x_1 and parameters η and σ^2 associated with the segment he can be ascribed to, is predicted as being log-normal with parameters $\hat{\eta}'x_1$ and $\hat{\sigma}^2$. The predicted F(w) are plugged in when estimating the structural model. The sample averages of the estimated expectation and standard deviation of the wage offer distribution equal 470 and 99 guilders per week, respectively. (Sample standard errors of these estimated values equal 86 and 27 guilders per week, respectively.)

In terms of the alternative interpretation of the structural model (see Subsection 2.2.2) the procedure described above does not give estimates of F(w) but instead it provides estimates of the individual distributions of vacancy wage offers corrected for wage competition (see equations (5) and (6)),

$$\frac{\int_0^w q_3(\omega)dG(\omega)}{\int_0^\infty q_3(\omega)dG(\omega)} \qquad w \geq 0$$

A final thing to note is that for a variety of reasons the current wage rate of an employed individual may exceed the wage rate that he obtained directly after becoming employed. In Section 2.6 a model that deals with this issue is considered. Further it is outlined how the wage offer distribution can be estimated in the presence of such wage differences.

2.5. Results

2.5.1. *Parameter estimates*

The parameter estimates for the structural model described in Subsections 2.2.1 and 2.3.3 are presented in Table 2. The unit time period is one week. For the age and education dummies the reference categories are the age category 46–64 and the level of education 1, respectively. Generally, the results seem to be in accordance with intuition. Education has a very significant influence on the job offer arrival rate. An individual having the highest level of education receives offers more than seven times as frequently as an individual with the lowest level of education. New entrants, having no experience, are offered jobs less often than experienced individuals. Being married is perceived by employers as a desirable property whereas being a head of a household is not. Single living individuals are also defined as being head of a household, so it may be that what really matters for employers is not the sheer presence of a partner but the presence of a family which makes the employee feel responsible. The importance of the number of working household members may be due to the fact that unemployed individuals for which this number is high have easier access to employers. However, it may also be a consequence of a positive correlation between unobserved characteristics of the unemployed individual and characteristics of other household members, as far as these characteristics are relevant for employers. The local unemployment rate has no significant influence on λ. Other indicators of the tightness of the labour market like the local UV ratio performed even worse. Van Opstal & Theeuwes (1986) who estimated a reduced-form duration model using Dutch data from 1984, also report this lack of significance. Presumably, job search is not restricted to a region anymore. Another explanation is that numbers on registered vacancies and unemployed individuals may not be accurate indicators of labour market tightness. Still, the estimate of -0.04 seems plausible: it implies that moving from the province with the highest rate of unemployment (24%) to the one with the lowest (15%) increases λ with a factor of almost 1.5.

The separate age coefficients in λ are not significant. Replacement of the age dummy variables by log(age) and its squared value results in even less significant estimates. However, a Likelihood Ratio test of the hypothesis that all age dummy coefficients equal zero leads to a rejection at the 10% level. In Section 2.3 it was noted that in some cases censoring of the forward recurrence time of young individuals may arise because they leave their parents' home in order to start working elsewhere. If so, then the coefficient on the age category 18-23 in the job offer arrival rate is under-estimated.

In terms of the alternative interpretation of the model (see Subsection 2.2.2) λ is the product of the vacancy arrival rate q_1 and the term q_4 which captures the influence of non-wage variables on the acceptance probability conditional on application q_2. We expect the unemployment rate, experience in previous jobs, education and age to be linked to q_1 while nationality and household characteristics probably are linked to q_4. The signs of the coefficients seem to confirm these prior expectations.

Turning to the rate of transition into nonparticipation, we see that being an unemployed new entrant has a positive effect on the exit rate out of the labour force (though not significantly), and that being unemployed and aged below 24 or over 45 also has a positive effect on this exit rate.

Table 2. Parameter estimates for the search model.

variable/parameter	coefficient	(t–ratio)
(i) *job offer arrival rate*		
constant	–6.08	(6.4)
Dutch	0.55	(1.3)
education: level 2	0.91	(3.3)
education: level 3	1.17	(3.6)
education: level 4	1.74	(2.8)
education: level 5	1.97	(2.8)
age category 18–23	0.68	(1.4)
age category 24–29	0.50	(1.2)
age category 30–45	0.16	(0.4)
new entrant	–0.82	(1.5)
head of household	–0.03	(0.1)
married	0.78	(2.5)
log (1 + # working in household)	1.03	(3.0)
local % unemployment rate	–0.04	(1.1)
(ii) *rate of transition into nonparticipation*		
constant	–4.91	(16.4)
age category 18–23	–0.41	(0.8)
age category 24–29	–1.06	(2.3)
age category 30–45	–1.39	(2.9)
new entrant	0.66	(1.4)
(iii) *disutility of unemployment*	0.74	(5.2)
Log–likelihood = –898.23		

The estimate of the disutility of unemployment v is smaller than one, implying that being unemployed is regarded as unpleasant. From the standard error of 0.14 it follows that the hypothesis v = 1 is rejected by a Wald test at the 10% level but not at the 5% level. However, the Likelihood Ratio test statistic for this hypothesis equals 20.4 $\gg \chi_1^2(0.95)$ so v = 1 is strongly rejected. This is in accordance with Narendranathan & Nickell (1985), who also found that v is significantly smaller that one. On the other hand, the estimate of v in Ridder & Gorter (1986) is not significantly smaller than one.

2.5.2. *The characteristics of the search process*

Given the parameter estimates, the main variables of the search process can be estimated and the influence of changes of the benefit level on these variables can be evaluated. We first present and interpret sample averages of the estimates. Subsequently the results are compared with other results in the literature and it is shown why our results differ in some respects. Table 3 presents sample averages of the estimates of λ, $\overline{F}(\phi)$ and ζ for different age categories and levels of education. The expected numbers of job offers and transitions into nonparticipation in a year can be obtained by multiplying the numbers in the λ and ζ row by 52.1. What strikes most is that in most cases $\overline{F}(\phi)$ is nearly equal to one. In particular those who are aged under 24 or over 46, or who have a primary education only, accept virtually every job that is being offered. Still, even individuals with a university degree have a probability of 0.8 of accepting the first job offered. It means that the reservation wages are located in the left part of the left tail of the wage offer distribution. The reason for this is the combination of on the one hand a very small job offer arrival rate and on the other hand very low values of the utility function in unemployment (v.u(b)) relative to employment (u(w)). Rejection of an offer may well imply a waiting time of more than a year before the next offer arrives. In the meantime the only source of income is benefits, which appear to be rather low relative to wages: the sample average of $\overline{F}(b)$ equals 0.9 and the ratio of the sample averages of b and E(w) equals 0.65. Moreover, because $v < 1$ there is a premium on being employed and one is willing to offer money for it by accepting lower-paid jobs. In fact, in our sample 79% of the unemployed even accept jobs with wages below their benefit level, that is, for these individuals $\phi < b$.

From Table 3 it can be inferred that for groups with a very low job offer arrival rate, almost 50% of all spells of unemployment end in a transition into nonparticipation. In other words, without such transitions the durations of unemployment for such individuals would be approximately twice as long. On the other hand, for the group of individuals with the highest level of education about 90% of all spells of unemployment end in a transition into employment.

Table 3. Probabilities and expectations.

(i) *by age category*

age category	18–23	24–29	30–45	46–64	average
λ (job offer arrival rate)	0.012	0.016	0.012	0.008	0.012
$\overline{F}(\phi)$ (proportion of offers acceptable)	0.99	0.94	0.96	1.00	0.97
ζ (rate of transition into nonparticipation)	0.007	0.003	0.002	0.007	0.004

(ii) *by level of education*

level of education	1	2	3	4	5
λ	0.004	0.014	0.018	0.024	0.033
$\overline{F}(\phi)$	1.00	0.98	0.94	0.89	0.82
ζ	0.004	0.004	0.004	0.004	0.003

The results so far enable us to investigate a number of questions related to the effectiveness of policies aimed at a reduction of unemployment durations. Table 4 presents for different age categories and levels of education sample averages of the elasticities of the reservation wage, the transition rate from unemployment into employment θ, and the expected duration d, with respect to the level of benefits. The results are unambiguous: a decrease in the level of benefits has virtually no effect on durations. Even for unemployed individuals with a university degree a 10% drop in benefits causes only a 1% drop in the expected duration. The individuals who suffer most from long spells (having primary education only, or aged under 24 or over 46) are completely insensitive to the benefits policy instrument. Note that elasticities refer only to infinitesimal changes. Still, even a large decrease in the level of benefits does not have much influence on duration. Individuals accept most jobs already, so a decrease in ϕ forced by a large decrease in b does not help much. The expected duration is bounded from below by $1/(\lambda+\zeta)$. From the results it is also clear that at an individual level additional educational training increases labour market opportunities.

Table 4. Elasticities with respect to benefits.

(i) *by age category*

age category	18–23	24–29	30–45	46–64	average
$\frac{\partial \log \phi}{\partial \log b}$ (reservation wage)	0.36	0.24	0.25	0.46	0.30
$\frac{\partial \log \theta}{\partial \log b}$ (hazard)	–0.01	–0.05	–0.04	–0.00	–0.03
$\frac{\partial \log d}{\partial \log b}$ (expected duration)	0.01	0.05	0.03	0.00	0.03

(ii) *by level of education*

level of education	1	2	3	4	5
$\frac{\partial \log \phi}{\partial \log b}$	0.44	0.24	0.23	0.19	0.16
$\frac{\partial \log \theta}{\partial \log b}$	–0.00	–0.03	–0.06	–0.07	–0.11
$\frac{\partial \log d}{\partial \log b}$	0.00	0.03	0.05	0.07	0.10

d equals the expected duration of unemployment.

The result that changing the benefits level has virtually no effect on the transition rate into employment and on the expected duration is in contrast with most empirical literature on unemployment durations. Early studies by Lancaster (1979), Nickell (1979), Lancaster & Nickell (1980) and Lancaster and Chesher (1983) suggest values of around 0.6 to 1.0 for the elasticity of the expected duration with respect to benefits. More recent work by Atkinson, Gomulka, Micklewright & Rau (1984), Narendranathan, Nickell & Stern (1985), Narendranathan & Nickell (1985) and Main & Shelly (1988) reports values of this elasticity that are typically ranging from about 0.1 to 0.3. The early studies use U.K. data from the beginning of the seventies while the later work uses more recent U.K. data. On the other hand, some studies that use Dutch data from the same observation period as we do (the mid–eighties), do not find any effect on duration of changing the benefits level. Van Opstal & Theeuwes

(1986) and Groot & ter Huurne (1988) obtain zero estimates for the elasticity of expected duration with respect to benefits from the estimation of reduced–form duration models, using data from young individuals only. Vissers & Groot (1989) estimate a series of reduced–form duration models: they consider several model specifications and several ways of defining the unemployment benefit variable, and they use different Dutch data sets from the mid–eighties to estimate the model. Nevertheless, their results are unambiguous in the sense that the elasticity of the transition rate into employment with respect to the benefits variable is insignificantly different from zero. Thus it seems that the results in Table 4 are not just an artefact of our particular sample but instead may be typical for The Netherlands in the mid–eighties.

In order to shed more light on this issue we examine in some detail the expression for the elasticity e of θ with respect to b. For simplicity we set ζ equal to zero. (Alternatively, ζ is put into ρ.) From equations (1) and (3), with u=log substituted in (3), it follows,

$$(11) \qquad e = \frac{\partial \log \theta}{\partial \log b} = - \left[\phi \cdot \frac{f(\phi)}{F(\phi)}\right] \frac{\rho v}{\rho + \theta}$$

with f being the derivative of F. One sees immediately that e depends on all other variables in the model. In other words, according to search theory cross–effects play a role in the effect that changing the benefits level has on duration. Consequently it is hard to regard the elasticity as a parameter that is equal in different economic environments. (See Feldstein & Poterba (1984) and Atkinson, Gomulka, Micklewright & Rau (1984) for similar statements.) Now let us have a look at the way e varies with the other variables. The second part of the right–hand side of (11) is an increasing function of ϕ. At first sight it seems that the way the term between brackets $k(\phi)$ varies with ϕ depends crucially on the class of wage offer distributions under consideration. However, it can be shown that for virtually every class of distributions for F, including the (truncated) normal, log–normal, Weibull, gamma, uniform, triangular, beta, (truncated) t, (truncated) logistic and log–logistic class of distributions, $k(\phi)$ is a strictly increasing function on the interval in $[0,\infty>$ on which f is positive, with $k(0) = 0$ (see Chapter 5). This implies that for almost every class of F, including the class we adopted, it holds that the smaller the reservation wage is, the smaller the effect of changing the benefits level on the transition rate into employment is. (The Pareto class of distributions is an extreme case because then $k(\phi)$ is a constant. This may explain the exceptionally high estimate of e in Ridder &

Gorter (1986), who estimated a structural model in which F is a Pareto distribution). Because the derivatives of ϕ with respect to b and v are positive, it therefore follows that the smaller the benefits level and the smaller the non-pecuniary utility of unemployment are, the smaller the effect of changing b on θ is. (This result is robust with respect to the functional form of u; for example, it can be shown that it also holds if u is linear. See also Feldstein & Poterba (1984) for some numerical examples in a simple model framework). Further, numerical calculations show that in the neighbourhood of the estimates of Tables 2 and 3 it holds that the smaller the job offer arrival rate is, the smaller the effect of changing b on θ is. In The Netherlands in the years around 1984 there was a very slack labour market. The level of unemployment was extremely high (the national unemployment percentage reached its peak in 1984 when it equalled 17.3%) while at the same time the number of vacancies was small (the V/U ratio was about 0.02 in 1984). Clearly labour market conditions were worse than the conditions that prevailed when the data for most of the previous studies mentioned before were collected. This shows up in the relatively low estimates of λ as reported in Table 3. Further, as shown before, in our data set the level of benefits is generally very small as compared to most of the wage offers. As a result, the estimated reservation wages are located in the left part of the left tail of F(w) and the elasticities of θ and the expected duration of unemployment with respect to b are almost zero. (Note that for $\zeta>0$ the latter elasticity is always smaller in absolute value than the former.)

A point that is related to the previous paragraphs concerns the influence of the sampling scheme on the sample averages in Tables 3 and 4. The data set used to estimate the model is basically a sample from the stock of the unemployed (on the date of the first interview). The distribution of the observed explanatory variables in the stock of the unemployed differs from that in the flow into unemployment, in the sense that values that are associated with a high expected duration are over represented in the stock (This can be inferred from the analytical results in Ridder (1984)). As a result, a comparison of sample averages of, for example, the estimates of λ and e obtained by using stock data (such as ours), with sample averages obtained by using flow data (such as those in Narendranathan & Nickell (1985)), is hampered by the fact that λ and e depend on observed explanatory variables that are unequally distributed in the two sampling schemes. Further, it follows that the results in Table 4 only refer to the effect that changes in b on average have on individuals in the stock of the unemployed. Note that in a reduced-form model framework such problems with regard to e do not exist because in reduced-form models e is a parameter that does not depend on explanatory variables.

In light of the previous paragraphs it is not surprising that the values of the elasticity of ϕ with respect to b are somewhat different from those found in other studies. Lancaster & Chesher (1983) and Narendranathan & Nickell (1985) suggest values between 0.1 and 0.2 for this elasticity. Main & Shelly (1988) report values of about 0.32 for youth training scheme participants and values of about 0.16 for other unemployed youths. In our sample the job offer arrival rate is extremely small so for an unemployed individual b is an important determinant of the expected discounted lifetime utility. Consequently the effect on ϕ of a change in b is relatively large. As explained above this does not translate into a substantial change in θ.

Obviously in the present context only micro effects of a cut in benefits can be investigated. On a macro level such a policy is likely to generate additional effects both on the inflow into unemployment and on the transition from unemployment into employment (Narendranathan, Nickell & Stern (1985)). Also, if there is an element of choice as to whether to become nonparticipant or not, then a cut in benefits may have an effect on ζ. The sign of this effect depends among other things on the dependence of the distribution of income of nonparticipants on the level of benefits. If benefits are decreased whereas the incomes of nonparticipants like conscripts and disabled remain unchanged then equation (2) does not hold anymore. Therefore an investigation of the relation between b and ζ should be made in a wholly structural model setting and is beyond the scope of this chapter. Inclusion of log (benefits) as a regressor in log ζ resulted in a highly insignificant parameter estimate of -0.14 ($t = 0.3$), all other things being almost identically equal.

Since the model does not allow for nonstationarity, it may be interesting to examine in what sense the results are affected by this omission. It is widely believed that the transition rate into employment θ is a decreasing function of duration. On the other hand, as we saw in Subsection 2.3.1, b may decrease during unemployment, and this makes θ ceteris paribus an increasing function of duration. One possible explanation for a decreasing θ is that the job offer arrival rate decreases sharply during unemployment, for example as a consequence of a scar effect of being unemployed for a long time, and that this decrease of λ offsets the increase in $\overline{F}(\phi)$. If θ is a decreasing function of duration then the expected duration of the backward and forward recurrence times exceeds the expected duration of completed durations of unemployment and a stock sample of unemployed individuals contains a relatively large number of long-term unemployed individuals. Further, if both b and λ decrease during unemployment then ϕ also decreases. So if nonstationarity is present in reality in the sense that b, λ, ϕ and θ all decrease, then b in the sample is on average smaller than the benefits level for the short–term unemployed and λ, ϕ and θ are under–estimated in the sense that shortly after the inflow into unemployment these variables are larger than estimated. From the discussion of equation (11) it then follows that in such cases the short–term unemployed are more sensitive with respect to changes in b than the results in Table 4 suggest, because in such cases e shortly after the inflow into unemployment is more negative than reported. However, short–term unemployed individuals may anticipate decreases of λ or b by modifying the reservation wage before these decreases take place. If such anticipations are strong then it is hard to elaborate on the effects that not allowing for nonstationarity may have on the estimates of the elasticity (see Chapter 3).

Another kind of nonstationarity is present if the transition rate into nonparticipation ζ increases as a function of duration, as a result of a discouraged worker effect, for example. By analogy from the argument pointed out above it may be expected that in such a case ζ is under–estimated for individuals who are long–term unemployed.

2.5.3. *The model specification revisited*

This subsection examines whether the results are sensitive with respect to changes in some of the assumptions. Changes in the way jobs are characterized in the model (infinite duration, constant wages) are referred to Section 2.6 where estimation results are presented for an extended model dealing with this.

In the structural model used for the empirical analysis v is the only exogenous variable which is estimated but not parameterized. It thus seems natural to extend the model by making v a function of observable individual characteristics. Also, one might ask why ρ is not estimated and why u is not parameterized, say, by assuming it to be a one–parameter CARA utility function. (CARA = constant absolute risk aversion; $u(x) = -\exp(-cx)$ with $c>0$.) Though such extensions do not raise identification problems in the statistical sense, it appears that there is not sufficient information in the data to be able to estimate such additional parameters. Apparently the likelihood is an almost completely constant function of such parameters in the neighbourhood of the optimum. This can be explained by recalling the results in Tables 3 and 4. First note that generally ϕ is small with respect to most wage offers, which implies that $f(\phi)$ is small so small changes in ϕ given values of λ, ζ and $F(w)$

do not affect the value of the likelihood function much. Secondly, u, ρ and v enter the likelihood only via ϕ. Therefore the correlation between estimates of parameters of u, v and ρ will be very high.

In the empirical model v is the only parameter that enters the likelihood via ϕ only. The discussion in the previous paragraph suggests that $\hat{v}$ might be biased if u is misspecified or if ρ has the wrong value. This is investigated by re-estimating the model with different u and ρ. Throughout the range of acceptable values of ρ the estimation results for λ and ζ hardly differ from the original results (which are obtained by assuming $\rho = 10\%$ per year.) The differences in the value of ρ are absorbed by $\hat{v}$, higher values of ρ resulting in higher values of $\hat{v}$ thus holding ϕ and therefore the fit of the model constant. For instance if $\rho = 5\%$ then $\hat{v} = 0.67$ (standard error: 0.19) while if $\rho = 15\%$ then $\hat{v} = 0.78$ (0.12). Still, $\hat{v}$ is always significantly smaller than 1 according to LR tests at the 1% level. Even in the limiting case of $\rho = \infty$ the estimate of v is significantly smaller than 1 ($\hat{v} = 0.91$).

We also tried to re-estimate the model using a linear utility function u of income. This did not work. In the process of maximizing the likelihood v tended to zero. This may be regarded as a justification for using a risk-averse specification of u because in that case the level of ϕ for $v = 0$ is ceteris paribus lower than the corresponding level in the risk-neutral case.

In Section 2.2 we stated the assumptions that equation (2) holds and that the non-pecuniary utility of being a nonparticipant equals that of being unemployed. In what sense are the results affected if these assumptions are relaxed? Denote the non-pecuniary component of utility in nonparticipation by v_1 and the corresponding component in unemployment by v_2. It can be shown that if $v_1 \neq v_2$ or $Eu(x) \neq u(b)$ then the parameter v in equation (3) has to be replaced by

$$\frac{\zeta . v_1 . \dfrac{Eu(x)}{u(b)} + \rho v_2}{\zeta + \rho} \tag{12}$$

in order to obtain the equation for the optimal reservation wage. So then $\hat{v}$ represents the estimate of expression (12). It follows that

$$v_1 . Eu(x) > v_2 . u(b) \Leftrightarrow \hat{v} > v_2$$

so if we believe that $v_1 > v_2$ or that $Eu(x) > u(b)$ then the estimate of v implies that the estimate of the disutility of unemployment is even smaller than 0.74.

In Section 2.3 we discussed the so-called memory problems. There it was argued that values of 3 and 9 months for τ_1 and t_1 respectively, were plausible. It appears that the parameter estimates are insensitive to changes of these values, though standard errors increase if τ_1 increases or t_1 decreases.

When deriving the distribution of the backward recurrence time t we assumed that the rate of entry into unemployment is constant until May 1984. One may question whether this assumption holds true. According to Pissarides (1986) in the U.K. the entry rate was fairly constant between 1967 and 1983 apart from an increase in 1979–1981. In the absence of reliable Dutch data we examine the sensitivity of the results with respect to the constant entry rate assumption by re-estimating the model with a time-varying entry rate. In particular we take as an alternative assumption that the entry rate q between January 1980 and January 1983 is twice as large as it is outside that time interval. In the appendix to this chapter the appropriate likelihood is derived. The main effect of the alternative assumption about q on the estimation results is that the exit rate out of unemployment $\theta + \zeta$ is estimated to be 13% larger. However, θ and ζ are still very small, and v, $\overline{F}(\phi)$ and the elasticities are insensitive to the change in the assumption on q. Thus, the main results and conclusions from Subsections 2.5.1 and 2.5.2 do not appear to be sensitive to changes in the assumptions about the time pattern of the entry rate into unemployment that are reasonable a priori.

One may question whether the estimation results are affected by a possible misspecification of the wage offer distribution which is estimated a priori. Obviously, F(w) plays a central role in the model because the trade-off between wages and benefits is a major determinant of search behaviour. We constructed F(w) which are log-normal and have same variances as before, but which have expectations that are shifted by 20% in comparison to the expectations derived in Section 2.4. Re-estimation of the model using these alternative F(w) resulted in values that are almost identical to those presented in Tables 2–4. The shifts in E(w) are absorbed by $\hat{v}$, a value of 1.2 times the original E(w) resulting in $\hat{v} = 0.82$ and a value of 0.8 times the original E(w) resulting in $\hat{v} = 0.64$. Consequently, the main conclusions are insensitive with respect to small misspecifications in the location of F(w).

When deriving the likelihood no account has been taken of unobserved heterogeneity in the sample. If unobserved heterogeneity is present in reality then the estimates may be inconsistent. However, estimating a structural model that allows for such heterogeneity is extremely complicated. For example, consider the case in which unobserved heterogeneity is present in λ. We may rewrite λ as a product.

$$(13) \qquad \lambda = \nu.\exp(x'\beta)$$

in which the random term ν represents unobserved heterogeneity in λ across the population. Substitution of equation (13) in equations (3) and (1) reveals that the hazard cannot be written explicitly as an explicit function of ν. Therefore, calculating the unconditional (on ν) duration density by integrating the conditional density with respect to the density of ν will be very complicated and is not pursued here.

2.6. An extended model

2.6.1. *The model*

In reality the duration of employment is not infinite, nor are wages constant during employment. The prospective rate of wage increases and the distribution of the duration of employment affect the value of search of an unemployed individual. Therefore they should be incorporated in the model. In this section we deal with this.

We assume that the duration of employment has an exponential distribution with parameter s which is the layoff rate. During one period of employment one can hold several consecutive jobs without intervening spells of unemployment. It is assumed that one returns to the state of unemployment if a layoff occurs, and that the duration of employment is stochastically independent of both the initial wage rate and the duration of unemployment that precedes employment.

During a spell of employment wages can increase for several reasons such as rising productivity or transitions from jobs with lower wages to jobs with higher wages without intervening spells of unemployment (on-the-job search). As a stylized description of this we assume that the wage pattern during employment is characterized by $w(t)$ giving the wage rate as a function of the time t that one is employed conditional on the initial wage $w(0)$.

$$(14) \qquad w(t) = w(0).e^{\alpha t}$$

in which α does not depend on $w(0)$ or t or on the duration of unemployment preceding employment. Though it is conceivable that mechanisms linking α, t and w(0) exist, the exploration of this is beyond the scope of the chapter.

The extensions of the model do not affect the stationarity property of search behaviour of the unemployed. In the appendix to this chapter it is proven that the reservation wage ϕ corresponding to the optimal strategy satisfies

$$(15) \qquad \log \phi = v.\log b + \frac{\lambda}{\rho+\zeta}\cdot\frac{\rho}{\rho+s}\cdot\int_{\phi}^{\infty} (\log w - \log \phi)dF(w) - \frac{\alpha}{\rho+s}$$

F(w) is the distribution of initial-wage offers, which is the distribution from which the w(0) are drawn. Note that the derivative of ϕ with respect to α is negative. If α is large then the value of search is high. However, this does not make the searcher more selective with regard to wage offers. It is profitable to give up more present income (a low w(0)) in order to obtain a higher income in the future.

The estimation of F(w) has to be reconsidered because in Section 2.4 we used a (cross-section) sample from the stock of the employed and therefore used data on current wages, that is, data on wages which are higher than the initial wages offered at the start of the current employment spell. We assume that the distribution of current wages is log-normal with parameters μ and σ^2. Thus, in Section 2.4 these parameters are estimated. The distribution F(w) of initial-wage offers has to be recovered from the distribution of current wages. In the appendix to this chapter it is shown that F(w) can be approximated by a log-normal distribution with parameters $(\mu+\log((s-\alpha)/s)$ and σ^2,

$$(16) \qquad F(w) \approx LN\ (\mu + \log \frac{s-\alpha}{s},\ \sigma^2)$$

This requires $s > \alpha$. The approximation is good for $s >> \alpha$.

2.6.2. *The results*

The approximation in equation (16) is used to obtain a priori estimates of the individual distribution functions F(w). The results from Section 2.4 provide the individual values of μ and σ^2. The parameter α is fixed at 4% per year. We used the elapsed duration of employment of individuals who were employed in April 1984 to estimate s. Since we assume that the entry rate into employment is constant (the stationarity assumption) these incomplete durations have an exponential distribution with parameter s. In accordance with the treatment of the memory problem in Subsection 2.3.2 durations are censored at 9 months. The ML estimate of s equals 14.4% per year (t-ratio equals 14.2) which implies that the expected duration of employment is almost seven years. This estimate may be biased for a variety of reasons (such as neglected unobserved heterogeneity) but we believe that for our purposes it is accurate enough.

From equation (16) it can be deduced that the expectation and the standard deviation of F(w) are $100.(\alpha/s)\% = 28\%$ smaller than those obtained in Section 2.4. The sample average of the probability that a random initial-wage offer exceeds the benefit level is 0.61 as opposed to 0.91 when F(w) is estimated as in Section 2.4.

The estimates and t-ratios of the parameters of λ and ζ hardly differ from those presented in Table 2. Further, the general pattern of the results presented in Tables 3 and 4 is preserved. Therefore only sample averages of the main variables are presented for the extended model (see Table 5).

Table 5. Estimates for the extended search model.

variable/parameter	coefficient	estimates for the basic model
v	0.83	0.74
λ	0.012	0.012
ζ	0.004	0.004
$\bar{F}(\phi)$	0.98	0.97
$\partial \log \phi/\partial \log b$	0.49	0.30
$\partial \log \theta/\partial \log b$	-0.04	-0.03
$\partial \log d/\partial \log b$	0.03	0.03

$\bar{F}(\phi)$, λ and ζ have almost the same sample averages as before. The parameter v is significantly smaller than 1 according to a LR test (test-statistic value $36.0 \gg \chi^2_1(0.99)$). The job offer acceptance probability is large because of the combination of a small job offer arrival rate and a low utility value attached to being in the state of unemployment. The latter holds both because one dislikes being unemployed for non-pecuniary reasons and because, in unemployment, income is constant whereas one expects it to increase in employment. In the extended model b is generally close to the median of F(w). So in this model it is the rate of income increases rather than the level of income which makes employment preferable from a material point of view. The elasticity of the expected duration with respect to the level of benefits is very small. The reasons for this are similar to those given in Subsection 2.5.2 to explain the results in Table 4.

In order to examine whether the estimates are sensitive to changes in the assumed values of ρ and α, the model is re-estimated for alternative values of ρ (5 and 15% per year) and α (3 and 5% per year). It appears that the differences in the values of ρ and α are absorbed by $\hat{v}$ and that all other results in Table 5 and the fit of the model are almost constant for the cases considered. The value of $\hat{v}$ ranges from 0.82 to 0.85 so the sensitivity of $\hat{v}$ to changes in the value of ρ is less than in the basic model.

In sum, the main conclusions from Section 2.5 about the parameter estimates, about the relative magnitudes of the main variables for different age categories and levels of education, and about the effects of changes in the level of benefits, remain unaffected. The results in this section suggest that on-the-job search may be an important factor for search behaviour of the unemployed. Therefore a topic for further research would be to extend the model to include on-the-job search explicitly. Using data of employed and unemployed individuals simultaneously, the wage offer distribution could be estimated along with the other variables. Also, some of the rather rigid assumptions that were made in this section could be relaxed in such a model.

2.7. Conclusions

In this chapter we have extended the existing empirical literature on structural job search models by specifying and estimating a model that allows for transitions from unemployment into nonparticipation. Moreover, a version of the model deals with the influence of prospective wage increases during employment on the search behaviour of the unemployed. The model is estimated using Dutch data from 1983–1985. The results indicate that almost every job offer is acceptable. The reason for this is the combination of a very small job offer arrival rate and low values of the utility function in unemployment relative to employment. If one turns down an offer then generally one has to wait for a very long time before the next offer arrives. In the meantime one is unemployed, which is disliked both for pecuniary and for non-pecuniary reasons. As for the pecuniary reasons, in the basic model these refer to the low level of benefits relative to wages. If account is taken of wage increases during employment then generally the estimated difference between benefits and initial–wage offers is much smaller. However, the prospect of wage increases causes the unemployed searcher to set a low reservation wage as well. The results imply that at an individual level a decrease in benefits is ineffective in reducing unemployment duration. The estimation results appear to be robust to varying certain assumptions.

Appendix to Chapter 2

2.A.1. Derivation of equation (3)

Basically, the derivation proceeds along the lines of Lancaster & Chesher's (1983) derivation of the reservation wage equation in a standard model with income maximization and $\zeta = 0$. First, consider a moment t at which an offer is pending. Let I_e denote the value at time t of following the optimal strategy. An acceptance policy can be characterized by a function p mapping $[0,\infty\rangle$ onto $[0,1]$ and giving for every w the probability that a wage offer w will be accepted. R is defined to be the return of rejecting the offer and behaving optimally afterwards. Because of the stationarity assumption I_e, p and R do not depend on t. Thus, at every moment at which an offer is pending, I_e denotes the present value of following the optimal strategy.

$$\text{(A1)} \qquad I_e = \sup_{p} \int_0^\infty \left[p(w).\frac{u(w)}{\rho} + (1-p(w)).R \right] dF(w)$$

It follows that the optimal acceptance policy p^* is given by

$$\text{(A2)} \qquad \begin{aligned} p^*(w) &= 1 && \text{if } u(w) \geq \rho.R \\ p^*(w) &= 0 && \text{otherwise} \end{aligned}$$

so p^* can be characterized by a reservation wage ϕ, satisfying

$$\text{(A3)} \qquad u(\phi) = \rho.R$$

Thus (A1) can be written as

$$\text{(A4)} \qquad I_e = R + \frac{1}{\rho}.\int_\phi^\infty u(w)-u(\phi)\ dF(w)$$

Let I_n denote the expected return at a moment at which a transition into nonparticipation occurs. From the assumptions on the expected utility during nonparticipation it follows that

$$\text{(A5)} \quad I_n = \int_0^\infty \int_0^\infty e^{-\rho t}.v.u(x)\ dt\ dH(x) = \frac{v.u(b)}{\rho}$$

in which H(x) is the c.d.f. of income flows of nonparticipants. Let $k(\tau)$ denote the p.d.f. of the distribution of the waiting time at t until the next event (job offer or transition into nonparticipation) occurs. Because of the stationarity assumption $k(\tau)$ does not depend on t and is distributed exponentially with parameter $\lambda+\zeta$. If an event occurs, the probability that this event is a job offer is equal to $\lambda/(\lambda+\zeta)$. Now R can be written as

$$\text{(A6)} \qquad R = \int_0^\infty k(\tau) \left[\int_0^\tau v.u(b).e^{-\rho s} ds + e^{-\rho t} \left\{ \frac{\lambda}{\lambda+\zeta}.I_e + \frac{\zeta}{\lambda+\zeta}.I_n \right\} \right] d\tau$$

which reduces to

$$\text{(A7)} \qquad R = \frac{v.u(b) + \lambda I_e + \zeta I_n}{\rho + \lambda + \zeta}$$

Substitution of (A3), (A4) and (A5) in (A7) gives the desired result. Note that for equation (3) to hold it is not necessary that the distribution of income flows of nonparticipants and the per-period utility function of nonparticipants are independent of the time spent in the state of nonparticipation. What is essential is that the expected discounted lifetime utility at the moment that one becomes a nonparticipant equals I_n equals $v.u(b)/\rho$. Therefore equation (A5) can be replaced by

$$\text{(A8)} \qquad I_n = \int_0^\infty e^{-\rho t} \int_0^\infty v(t).u(x;t)\ dH(x|t)\ dt = \frac{v.u(b)}{\rho}$$

in which t denotes the duration in the state of nonparticipation; the definitions of v(t), u(x;t) and H(x|t) are obvious.

The model can also be extended in another direction without changing the outcomes. From the examples of transitions from unemployment into nonparticipation it is clear that one can also expect transitions from employment into nonparticipation to be present in reality. If so then the unemployed individual can be assumed to take account of this when determining his optimal strategy. Let transitions from employment into nonparticipation arrive according to a Poisson process with arrival rate ω. We assume that the expected discounted lifetime utility at the moment that one becomes a nonparticipant is independent of the origin state and is denoted by I_n. It can be proven that, instead of equation (3), the reservation wage satisfies

$$\text{(A9)} \qquad u(\phi) = \frac{1}{\rho+\zeta}.\left\{ \rho I_n.(\zeta-\omega) + v.u(b).(\rho+\omega) \right\} + \frac{\lambda}{\rho+\zeta} . \int_\phi^\infty u(w)-u(\phi)\ dF(w)$$

If we impose that $\omega = \zeta$, that is, if we assume that the transition rate into nonparticipation is the same for employed and unemployed individuals, then equation (A9) reduces to equation (3). This result holds regardless of the value of I_n as long as it is fixed. For our purposes it is even more interesting that if equation (A8) is substituted in equation (A9), this equation again reduces to equation (3). That is, the reservation wage does not depend on ω if (A8) holds.

2.A.2. Likelihood function in case of a time-varying entry rate

If the entry rate into unemployment is dependent on time then the backward recurrence time t no longer has an exponential distribution. Consequently the likelihood contribution L_2 (see equation (10)) has to be modified. From Ridder (1984), the density function $h(t|x)$ of t given time-independent personal characteristics x is given by

$$\text{(A10)} \qquad h(t|x) = \frac{q(-t|x).e^{-\omega t}}{\int_0^\infty q(-s|x).e^{-\omega s} ds} \qquad t \geq 0$$

in which $\omega \equiv \omega(x) \equiv \theta(x) + \zeta(x)$ and in which $q(-t|x)$ is the entry rate at t units of time before April 1984. In Subsection 2.5.3 it is assumed that

$$\text{(A11)} \qquad \begin{aligned} q(-t|x) &= q(0|x) \qquad && 0 \leq t < t_2 \text{ or } t \geq t_3 \\ q(-t|x) &= 2q(0|x) \qquad && t_2 \leq t < t_3 \end{aligned}$$

with t_2 and t_3 equal to 16 and 52 months, respectively. The variable t is censored at t_1 (see Subsection 2.3.2). By substituting (A11) in (A10), taking account of the censoring, and by taking the logarithm, the modified L_2 is obtained. This expression does not depend on $q(0|x)$.

2.A.3. Derivation of equation (15)

The line of argument and the notation of Appendix 2.A.1 are followed. Equations (A7) and (A8) remain valid. Equation (A1) is replaced by

$$\text{(A12)} \qquad I_e = \sup_p \int_0^\infty \left[p(w).E_t \left\{ \int_0^t e^{-\rho\omega}.u(e^{\alpha\omega}.w) d\omega + e^{-\rho t}.R \right\} + (1-p(w)).R \right] dF(w)$$

The expectation E_t is taken w.r.t. the duration of employment. The reservation wage ϕ characterizes the optimal strategy,

$$(A13) \qquad E_t \int_0^t e^{-\rho\omega}.u(e^{\alpha\omega}.\phi)d\omega = E_t(1-e^{-\rho t}).R$$

Substitution in (A12) gives, noting that u is the logarithmic function,

$$(A14) \qquad I_e = R + \frac{1}{\rho}.E_t(1-e^{-\rho t}) . \int_\phi^\infty (\log w - \log \phi)dF(w)$$

Equation (A13) can be simplified to

$$(A15) \qquad E_t \int_0^t \alpha\omega e^{-\rho\omega}d\omega = E_t(1-e^{-\rho t}).\left[R - \frac{\log \phi}{\rho}\right]$$

Because t ~ exponential (s) it holds that

$$E_t(1-e^{-\rho t}) = \frac{\rho}{\rho+s}$$

$$E_t \int_0^t \alpha\omega e^{-\rho\omega}d\omega = \frac{\alpha}{(\rho+s)^2}$$

so equations (A14) and (A15) can be simplified to

$$(A16) \qquad I_e = R + \frac{1}{\rho+s} . \int_\phi^\infty (\log w - \log \phi)dF(w)$$

$$(A17) \qquad \log \phi = \rho.R - \frac{\alpha}{\rho+s}$$

Substitution of (A16), (A17) and (A8) in (A7) gives the desired result.

2.A.4. Approximation of the distribution of initial-wage offers

In order to avoid confusion between initial wages and current wages the latter are denoted by y and the former by w. The distribution over the population of completed durations of employment is exponential with parameter s. We observe a (cross-section) sample from the stock of the employed, which means that the durations of employment t are incomplete. However, the entry rate into employment is time-independent due to the stationarity assumption. Therefore such incomplete durations have an exponential distribution with parameter s as well.

An observed (current) wage y is the product of two unobserved stochastic terms

$$\text{(A18)} \qquad y = e^{\alpha t}.w$$

in which t denotes the incomplete duration of employment. Since t and w are independent, the moments of y are easily expressed in terms of the moments of w,

$$\text{(A19)} \qquad E(y) = E(e^{\alpha t}).E(w) = \frac{s}{s-\alpha}.E(w)$$

$$\text{(A20)} \qquad var(y) = (\frac{s}{s-\alpha})^2.var(w) + (\frac{\alpha}{s-\alpha})^2.\frac{s}{s-2\alpha}.E(w^2)$$

Define $\xi = \alpha/s$. Equations (A19) and (A20) can be rewritten as

$$E(w) = (1-\xi).E(y)$$

$$var(w) = (1-\xi)^2.var(y) + o(\xi)$$

Consequently, if we use

$$\text{(A21)} \qquad w = (1-\xi).y$$

in order to recover F(w) from the distribution of y, then the first moment of the distribution thus obtained is correct while the second central moment is correct up to the second order of α/s. For α small as compared to s the distribution of w based on equation (A21) is a good approximation of the true F(w) though the variance of F(w) is somewhat overstated. It is assumed that y $\sim LN(\mu,\sigma^2)$ so

$$\text{(A22)} \qquad (1-\xi).y \sim LN(\mu+\log(1-\xi),\ \sigma^2)$$

Therefore F(w) is approximated according to equations (A21) and (A22). Note that from equation (A18) and from the assumptions on the parametric forms of the distributions of y and t, F(w) could be deduced exactly. However this gives rather problematic results for the parametric forms that were chosen. Rather than modifying these choices we prefer to approximate F(w) as set out in the previous paragraph.

CHAPTER 3

NONSTATIONARITY IN JOB SEARCH THEORY

3.1. Introduction

This chapter examines the movement of a job seeking individual's reservation wage over time in a general nonstationary job search model. Also, results concerning comparative dynamics of the reservation wage and the distribution of the duration of unemployment are derived. As an empirical illustration a nonstationary structural model is estimated. The nonstationarity originates from the decrease of the level of benefits when unemployment duration equals two years. From the results some detailed policy recommendations can be deduced, as one is able to distinguish the effect of a change of the level of benefits in the first two years of unemployment from the effect of a change of the level after that period.

Recently the use of job search models for the analysis of unemployment duration has become widespread. The reduced-form approach in empirical studies (see for example Lancaster (1979)) in which only the hazard of the duration distribution is estimated seems to be replaced gradually by a more structural approach. The latter way of modelling is characterized by the explicit use in empirical analysis of the reservation wage equation as stated by the theory. For example Yoon (1981), Lancaster & Chesher (1983), Lynch (1983), Narendranathan & Nickell (1985) and van den Berg (1990c) use the complete theoretical framework of job search theory to make inferences about search behaviour.

However, the structural models used in these studies are stationary. This implies that variables like unemployment benefits or the rate of arrival of job offers are assumed to be constant over the spell of unemployment, which is often at variance with reality. What's more, various reduced-form empirical studies indicate a significant duration dependence of the re-employment probability, which is generally interpreted as evidence in favor of the presence of nonstationarity (see e.g. Blau & Robins (1986b), Kooreman & Ridder (1983), Lancaster (1979) and Narendranathan, Nickell & Stern (1985)). Consequently there is a need to model reservation wage movements over time based on a nonstationary theoretical framework.

In the last two decades a few papers have been published that pay some attention to nonstationarity in job search theory (see e.g. Burdett (1979), Gronau (1971), Heckman & Singer (1982), Lippman & McCall (1976b) and Mortensen (1986)). Though these articles draw important qualitative conclusions concerning the movement of the reservation wage over time, generally no attention is paid to a rigorous derivation of formulae for the time dependence of the reservation wage. Furthermore, only very specific departures from stationarity are examined, like finite lifetimes or shifting wage offer distributions. Most models are specified in discrete time which means that an empirical implementation would require a arbitrary choice concerning the length of the unit time interval. These remarks also apply to Wolpin (1987) who estimates a structural model that allows for duration dependence of the job offer arrival rate. Kiefer & Neumann (1979b) estimate a discrete-time search model in which exactly one job offer per period is assumed to arrive and in which the reservation wage is a linear function of some explanatory variables including unemployment duration. This linear specification is not derived from theory so the model might be called semi-structural. It appears that duration has a significant negative influence on the reservation wage though it is not clear which economic causes should be held responsible for this effect.

In this chapter we examine the consequences of nonstationarity in continuous-time job search models in a rather general setting. Section 3.2 gives a brief overview of job search theory. Various causes of nonstationarity that may arise are discussed, like macro-economic events and changes in the personal situation of individuals during the spell of unemployment. In Sections 3.3 and 3.4 we present the main theorems concerning the movement of the reservation wage over time in nonstationary models. The exogenous variables like unemployment benefits and the wage offer distribution are allowed to vary over time in a very general way. The more specific the assumptions about the time paths of the exogenous variables, the more detailed are our inferences about the time path of the reservation wage. In Section 3.3 we also give some comparative dynamics results. These results concern the shift in the optimal reservation wage path if we replace some particular time path of an exogenous variable by another. We also examine the unemployment duration density in the case of nonstationarity.

In Section 3.5 we illustrate by means of an empirical example the importance of allowing for nonstationarity. In The Netherlands in the beginning of the eighties the benefits level during the first years of unemployment is related to the pre-unemployment wage while the level after that is determined by the public assistance system. As a consequence benefits generally decrease substantially when duration equals about 2 years. In a nonstationary structural model one can analyze in detail the effects of these changes, not only on the expected duration but also on the optimal reservation wage path. Using survey data on unemployed individuals from 1983, a nonstationary continuous-time structural job search model that allows for such changes is estimated. Given the parameter estimates we calculate the elasticities of the expected duration with respect to the level of benefits before and after 2 years of unemployment. It appears that for most individuals the elasticity of duration with respect to the level of benefits after 2 years is much larger than the elasticity of duration with respect to the level before 2 years. Section 3.6 concludes.

3.2. Job search theory and the introduction of nonstationarity

Job search theory tries to describe the behaviour of unemployed individuals in a dynamic and uncertain world. Job offers arrive at random intervals following a (non-homogeneous) Poisson process with arrival rate λ. Such job offers are random drawings (without recall) from a wage offer distribution with distribution function $F(w)$. Every time an offer arrives the decision has to be made whether to accept the offer or reject it and search further. Once a job is accepted it will be kept forever at the same wage. It is assumed that individuals know λ and $F(w)$ but that they do not know in advance when job offers arrive and what wages are associated with them. During the spell of unemployment, unemployment benefits b are received. Unemployed individuals aim at maximization of their own expected present value of income (over an infinite horizon).

The job search model described here contains three exogenous variables (λ, b and $F(w)$) and one constant parameter ρ which is the subjective rate of discount. For expositional purposes the theoretical results in this chapter are stated in terms of this basic model. At the end of Section 3.4 it is outlined how the results can be generalized to a setting that is more realistic with regard to the function that is to be maximized and also with regard to the process of search. We now discuss the concepts of stationarity and nonstationarity in the basic model. Let time T_0 denote the point of time at which an individual becomes unemployed. We call the job search model that describes the search behaviour of this individual stationary if the exogenous variables λ, b and $F(w)$ are constant on the time interval $[T_0,\infty\rangle$ and do not depend on realizations of offer times or wage offers. In combination with the infinite horizon assumption this means that in the case of stationarity the unemployed individual's perception of the future is independent of time or unemployment duration. Consequently, the optimal strategy is constant during the spell of unemployment. Let us assume that $F(w)$ is continuous in w, that this distribution has a finite first moment and that $0 \leq \lambda < \infty$, $0 < \rho < \infty$ and $0 \leq b < \infty$. For a stationary job search model satisfying these conditions it has been shown many times that the optimal strategy can be characterized by the reservation wage property (see e.g. Lancaster & Chesher (1983)). A job offer is acceptable if its wage exceeds the reservation wage ϕ while a wage below ϕ induces one to reject the offer and search for a better one. The reservation wage is the unique finite solution of

$$(1) \qquad \phi = b + \frac{\lambda}{\rho} \int_{\phi}^{\infty} (w-\phi) \; dF(w)$$

Nonstationarity arises if one or more of the exogenous variables change after T_0. Such a change may be due to business cycle effects. For instance, an increase in the aggregate unemployment level may induce a fall in λ. Changes may also occur because of policy changes like a reduction of all unemployment benefits. Finally, for a job searcher the exogenous variables may change because of changes in his personal situation. Unemployment benefits and λ may be dependent on the elapsed unemployment duration. Sooner or later these features of the labour market and personal characteristics of job searchers are recognized and used in determining the optimal strategy. So, generally, the optimal strategy is not constant in the case of a nonstationary model.

In this chapter we consider nonstationarity as a result of the time dependence and duration dependence of exogenous variables. Dependencies of exogenous variables on the number of rejected offers or the levels of wages associated with rejected offers are ruled out. Further, throughout the chapter we will be concerned with job searchers with perfect foresight in the sense that they are assumed to correctly anticipate changes in the values of the exogenous variables. In other words, we expect people to know how the exogenous variables are related to unemployment duration. In Section 3.6 we turn to the issue of relaxing this assumption.

3.3. The reservation wage in nonstationary job search models

3.3.1. *Assumptions*

For ease of exposition we let calendar time start at the moment that one becomes unemployed, so that calendar time and unemployment duration coincide. In this way duration dependence and other forms of nonstationarity can be considered simultaneously. In order to obtain properly defined present values and in order to restrict attention to economically meaningful cases, the following weak assumptions concerning the exogenous variables and ρ are made.

1. *Wage offers at time* t *are drawn randomly from a distribution with a distribution function* $F(w;t)$, *which is a continuous function of* w *and strictly monotonically increasing in* w *on some interval* $<\alpha(t),\beta(t)>$ *with* $0 \le \alpha(t) < \beta(t) \le \infty$, $F(\alpha(t);t) = 0$ *and* $\lim_{w \to \beta(t)} F(w;t) = 1$ *for every* $t \ge 0$. *The mean of the distribution is a uniformly bounded function of* t.
2. *For every* $t \ge 0$, $0 \le \lambda(t) \le K < \infty$ *and* $0 \le b(t) \le K < \infty$; K *being a fixed number.*
3. $F(w;t)$, $\lambda(t)$ *and* $b(t)$ *are continuous functions of* t *on* $[0,\infty>$ *except possibly for a finite number of points. If an exogenous variable is discontinuous in* t *at some point, say* t^*, *then it is right-continuous, and the left-hand limit of this variable at* t^* *does exist (e.g. in the case of* b: $\lim_{t \downarrow t^*} b(t) = b(t^*)$ *and* $\lim_{t \uparrow t^*} b(t)$ *exists).*
4. *There exists some number* T *such that all exogenous variables are constant on* $[T,\infty>$.
5. $0 < \rho < \infty$.

Note that a model which satisfies Assumptions 1–5 allows for quite general patterns of movement of the exogenous variables over time, comprising virtually every nonstationary situation that may arise in practice.

3.3.2. *The optimal path of the reservation wage*

We now present a characterization of the time path of the optimal strategy.

<u>Theorem 1.</u>

Let Assumptions 1–5 be satisfied. Then the optimal strategy of a job searcher can be characterized by a reservation wage function $\phi(t)$ *giving the reservation wage at time* t. $\phi(t)$ *is a unique, bounded and continuous function* *of* t *and it satisfies the following differential equation for every point in time at which* b(t), λ(t) and F(w;t) *are continuous in* t.

$$(2) \qquad \phi'(t) = \rho.\phi(t) - \rho.b(t) - \lambda(t)\,.Q(\phi(t);t)$$

where $Q(\phi(t);t)$ *is defined as*

$$Q(\phi(t);t) = \int_{\phi(t)}^{\infty} (w-\phi(t))dF(w;t) = \int_{\phi(t)}^{\infty} \overline{F}(w;t)dw \textit{ with } \overline{F}(w;t) = 1-F(w;t)$$

If one or more of the exogenous variables are discontinuous in t *at some point, then the right-hand side of* (2) *gives the right-hand derivative of* ϕ *with respect to* t *at that point. The left-hand derivative can be calculated by replacing the values of the exogenous variables at* t *in the right-hand side of* (2) *by their left-hand limits at that discontinuity point.*

The proof is in the appendix.

The differential equation (2) is also given by Mortensen (1986). However, in Mortensen's model the exogenous variables are forced to have very simple functional forms; in fact the only departure from stationarity is (in terms of our model) a simultaneous discrete change in λ and b when the unemployment duration equals T time-units. This change is interpreted to be a consequence of liquidity constraints.

In order to get an intuitive feeling for equation (2) we rewrite it in terms of the optimal present value of search R(t) at time t. From the appendix, $\phi(t) = \rho.R(t)$ so $\phi(t)$ is the wage rate which makes the individual indifferent between working and being unemployed at t. It follows,

$$(3) \qquad \rho R(t)dt = \frac{\partial R}{\partial t}dt + b(t)dt + \lambda(t)dt \,.\int_{\phi(t)}^{\infty} \left[\frac{w}{\rho} - R(t)\right] dF(w;t)$$

Suppose the optimal value R is an asset which can be traded in a perfect capital market with an interest rate that equals the discount rate ρ. In equilibrium the return from the asset value in a small time interval [t,t+dt>, which is $\rho R(t)dt$, must equal what one expects to get from holding the asset in that period. The latter consists of three parts: first, the appreciation of the asset value in the time interval; second, the benefits flow in the interval; and third, the expected gain of finding a job during the period (see Pissarides (1985) for other examples of such an interpretation).

Another way to look at equation (2) requires the introduction of a function $\phi_0(t)$, giving the optimal reservation wage at time t if the environment remains stationary after t, i.e. from equation (1), $\phi_0(t)$ is the unique finite solution of

$$(4) \qquad \phi_0(t) = b(t) + \frac{\lambda(t)}{\rho} \cdot Q(\phi_0(t);t) \qquad\qquad t \geq 0$$

Suppose we want to compare $\phi(t)$ and $\phi_0(t)$. Of course, $\phi(t) = \phi_0(t)$ implies $\phi'_R(t) = 0$ (a subscript R denotes a r.h.d.). Further, from equation (2), for every $\phi(t)$,

$$(5) \qquad \frac{\partial \phi'_R(t)}{\partial \phi(t)} = \rho + \lambda(t)\overline{F}(\phi(t);t) = \rho + \theta(t) > 0$$

in which $\theta(t)$ denotes the exit rate out of unemployment at time t (see Subsection 3.3.4). Consequently,

$$(6) \qquad \phi(t) \lesseqgtr \phi_0(t) \;\; \Leftrightarrow \;\; \phi'_R(t) \lesseqgtr 0$$

Let $R_0(t)$ denote the optimal value of search at t in case $\phi_0(t)$ is the optimal reservation wage. It is clear that $\phi_0(t) = \rho.R_0(t)$. Using $R_0(t)$ and $R(t)$ it can be shown that relationship (6) is perfectly plausible. If for example $\phi(t)>\phi_0(t)$ then $R(t)>R_0(t)$ which means that there are future changes in the values of the exogenous variables that altogether benefit the value of search $R(t)$ as compared to the 'stationary state' value of search $R_0(t)$. As time proceeds, these future changes come nearer. Both because future income is discounted by a positive rate ρ and because the probability of not finding a job before the changes take place (following the optimal strategy) increases as time proceeds, this implies that R will rise at t (compare equation (5)). So the right-hand derivative of R with respect to time at t is positive and consequently $\phi'_R(t) > 0$. Note that the argument applies to every two possible reservation wages at t, in the sense that it makes clear that given the values of the exogenous variables at t, $\phi_1(t) > \phi_2(t)$ implies $\phi'_{1_R}(t) > \phi'_{2_R}(t)$. In Section 3.4, where we make an additional assumption concerning the exogenous variables, we return to the interrelations between ϕ, ϕ' and ϕ_0.

Theorem 1 can be used in order to determine ϕ as a function of time. First solve for ϕ at the point T after which all exogenous variables are constant (this is easily done using equation (1)). $\phi(t)$ is a continuous function of t. Therefore $\phi(T)$ serves as an initial condition for the differential equation (2) in the time interval ending at T within which the exogenous variables are continuous. Thus $\phi(t)$ can be solved for every t in this interval. Backward induction leads to the solution $\phi(t)$ for every $t \geq 0$.

If restrictions are placed on the way that the exogenous variables may vary over time, then sometimes qualitative conclusions can be drawn concerning the time path of ϕ. In the remainder of this subsection sufficient conditions are given for the reservation wage to be strictly decreasing. Consider models in which one of the exogenous variables is time dependent in a way that is described by one of the following four cases, while the others are constant on the interval $[0,\infty\rangle$.

K_1) $\forall t \in [0,T\rangle,\ \forall \tau > 0,\ b(t) > b(t+\tau)$.

K_2) $\forall t \in [0,T\rangle,\ \forall \tau > 0,\ \lambda(t) > \lambda(t+\tau)$.

K_3) $\forall t \in [0,T\rangle,\ \forall \tau > 0$, $F(w;t)$ *first-order stochastically dominates* $F(w;t+\tau)$, *that is*, $\forall w \in \langle \alpha(t+\tau), \beta(t) \rangle,\ \overline{F}(w;t) > \overline{F}(w;t+\tau)$.

K_4) $\forall t \in [0,T\rangle,\ \forall \tau > 0$, $F(w;t)$ *is a mean-preserving spread of* $F(w;t+\tau)$, *that is*, $E(w;t) = E(w;t+\tau)$ *and*
$$\forall x \in \langle \alpha(t), \beta(t) \rangle, \quad \int_{\alpha(t)}^{x} F(w;t)dw > \int_{\alpha(t)}^{x} F(w;t+\tau)dw.$$

Note that in all cases we allow the exogenous variable to be discontinuous in a finite number of points. In order to rule out uninteresting situations in which decreasing exogenous variables do not make the reservation wage time dependent, we impose the restrictions that in case K_2 for every $t \in [0,T\rangle$, $\phi(t) < \beta(t)$ has to hold, whereas in case K_3 for every $t \in [0,T\rangle$, $\phi(t) < \beta(t)$ and $\lambda > 0$ have to hold and in case K_4 for every $t \in [0,T\rangle$, $\alpha(t) < \phi(t) < \beta(t)$ and $\lambda > 0$ have to hold. These restrictions can be characterized by the following restrictions on the exogenous variables in each case,

K_2) $b < \beta$.

K_3) $b < \beta_L(T),\ \lambda > 0$.

K_4) $\alpha_L(T).\left[1 + \dfrac{\lambda}{\rho}\right] - \dfrac{\lambda}{\rho}.E(w;T) < b < \beta_L(T),\ \lambda > 0$.

in which $f_L(a)$ denotes the left-hand limit of $f(x)$ at $x=a$ if it exists. Note that if for every $t \geq 0$, $\lambda(t) > 0$, $\alpha(t) = 0$ and $\beta(t) = \infty$ then these restrictions are always satisfied. Also note that a decreasing location or scale of the wage offer distribution are special cases of K_3 and K_4, respectively.

Theorem 2.

Let Assumptions 1-5 *be satisfied. In addition, let one exogenous variable be* *time dependent according to* K_1, K_2, K_3 *or* K_4 *while the others are constant on the time interval* $[0,\infty\rangle$. *Then*

(i) $\forall t\in[0,T\rangle$, $\phi(t) < \phi_0(t)$ *with* $\phi_0(t)$ *as defined in equation* (4).

(ii) $\forall t\in[0,T\rangle$, $\phi'(t) < 0$ *if this derivative exists. At points* t *where* $\phi'(t)$ *does not exist (i.e. points at which one of the exogenous variables is discontinuous), both* $\phi_L'(t) < 0$ *and* $\phi_R'(t) < 0$ *hold. If an exogenous variable is discontinuous at* T, *then* $\phi_L'(T) < 0$, *otherwise* $\phi'(T) = 0$.

The proof is in the appendix. Note that simultaneous occurrence of some K_1, K_2, K_3, K_4 can be examined by sequential application of Theorem 2. Lippman & McCall (1976b) consider a generalization of case K_3 for which they derive a result similar to Theorem 2 in a discrete time model in which exactly one job offer arrives per period.

Clearly, the results make economic sense. Any future decrease in b, λ or the mean or variance of F will make the value of search in the present smaller than it would have been if the exogenous variables were constants. From the discussion of equations (5) and (6) this means that $\phi(t) < \phi_0(t)$ for every $t\in[0,T\rangle$ and that ϕ decreases as lower values of the exogenous variables come nearer.

In the basic job search model that is described in Section 3.2 and Subsection 3.3.1 it is assumed that once a job offer is accepted it will be held forever. The model equations become intractable if one tries to relax this assumption by allowing individuals to quit or to be laid off, because nonstationarity in future spells of unemployment influences the optimal strategy in the present spell. However, Burdett & Sharma (1988) argue that the no–quits assumption is unduly strong if there is duration dependence in unemployment according to K_1. Basically, the argument is that rejecting a job offer is sub–optimal to accepting it and quitting immediately thereafter, because in the latter case one starts with a fresh spell of unemployment and as a result one obtains a higher level of benefits. However, such behaviour is very unlikely to occur in practice since generally there are effective legal barriers that discourage individuals to act that way. For instance in The Netherlands individuals who quit voluntarily do not get any benefits at all. Moreover, usually the level of benefits in the first period of unemployment is positively related to the pre–unemployment wage if one has had a job before becoming unemployed. Since a model in which quits and layoffs are allowed and in which each and every feature of the benefits system is incorporated would be too complicated to analyze we prefer to stick to the assumption that jobs are held forever.

3.3.3. *Comparative dynamics*

In this subsection we examine the consequences for the optimal reservation wage path when replacing some particular time path of an exogenous variable by a different (higher) path. For sake of convenience we will be using the term 'reference model' in case every exogenous variable follows the reference path, while the term 'alternative model' denotes cases in which one exogenous variable does not follow its reference path while the others do. Variables in the reference model will be labelled with a subscript r. Consider two arbitrary points in time t_1 and t_2, such that $0 \leq t_1 < t_2 \leq \infty$. We consider four different departures from the reference model:

C_1) $\quad \forall t \in [t_1, t_2\rangle$, $b(t) > b_r(t)$.

C_2) $\quad \forall t \in [t_1, t_2\rangle$, $\lambda(t) > \lambda_r(t)$.

C_3) $\quad \forall t \in [t_1, t_2\rangle$, $F(w;t)$ *first-order stochastically dominates* $F_r(w;t)$, *that is*, $\forall w \in \langle \alpha_r(t), \beta(t)\rangle$, $\overline{F}(w;t) > \overline{F}_r(w;t)$.

C_4) $\quad \forall t \in [t_1, t_2\rangle$, $F(w;t)$ *is a mean-preserving spread of* $F_r(w;t)$, *that is*, $E(w;t) = E_r(w;t)$ *and*

$$\forall x \in \langle \alpha(t), \beta(t)\rangle, \quad \int_{\alpha(t)}^{x} F(w;t)dw > \int_{\alpha(t)}^{x} F_r(w;t)dw.$$

It is important to remark that in every case above, for every exogenous variable the time paths in the reference model and the alternative model are equivalent outside the interval $[t_1, t_2\rangle$. Notice that changing the location and scale of the wage offer distribution are special cases of C_3 and C_4, respectively.

Theorem 3.

Consider one of the deviations C_1, C_2, C_3 *or* C_4 *from a reference model. Let the exogenous variables of both the reference model and the alternative model satisfy Assumptions* 1–5. *In addition, assume that in cases* C_2, C_3 *and* C_4 *there is a* $t_3 \in [0, t_2\rangle$ *such that* $\forall t \in [t_3, t_2\rangle$, $\phi_r(t) < \beta(t)$, *while in case* C_4 *also* $\forall t \in [t_3, t_2\rangle$, $\phi_r(t) > \alpha(t)$. *Moreover, in cases* C_3 *and* C_4 $\forall t \in [t_3, t_2\rangle$, $\lambda(t) > 0$ *has to hold. Then, as a result,*

(i) $\quad \forall t \in [0, t_2\rangle$, $\phi(t) > \phi_r(t)$.

(ii) $\quad \forall t \in [t_2, \infty\rangle$, $\phi(t) = \phi_r(t)$.

(iii) $\quad \forall t \in [0, t_1\rangle$, $\phi'(t) > \phi_r'(t)$ *if* t *is a point at which* ϕ *and* ϕ_r *are* *differentiable with respect to time. If they are not differentiable at some point* $t \in [0,t_1>$ *then the inequality still holds in that point for the left- and right-hand derivatives. Further,* $\phi'_L(t_1) > \phi_{rL}'(t_1)$. *(A subscript* L *denotes left-hand derivatives.)*

(iv) $\phi'_L(t_2) \leq \phi_{rL}'(t_2)$.

The proof is given in the appendix. By reversing the reference model and the alternative model, we obtain the results in case of 'downward' shifting exogenous variables. Simultaneous occurrence of some C_1, C_2, C_3, C_4 can be examined by sequential application of Theorem 3. In Theorem 3, the inequality restrictions concerning $\phi_r(t)$ and $\lambda(t)$ are imposed only for expositional elegance; they rule out uninteresting cases in which changing exogenous variables do not influence the reservation wage path. Sufficient conditions in terms of the exogenous variables are given in the appendix. If for every $t>0$ $\alpha(t)=0$, $\beta(t)=\infty$ (which holds for example in case of log-normally distributed wages) and $\lambda(t)>0$, then the restrictions are always satisfied.

The intuition behind (i) and (ii) is straightforward. Any future shift in the time path of exogenous variables that benefits the expected discounted lifetime income induces job searchers to be more selective in their search process. As for the period up to t_1, the shift in exogenous variables after point t_1 becomes more important when going forward in time. This implies that $\phi(t)$ shifts away from $\phi_r(t)$ when t comes closer to t_1. However, it is not always true that $\forall t \in <t_1,t_2>$, $\phi'(t) < \phi'_r(t)$, if properly defined. It is easy to find time paths of the exogenous variables in the alternative model that cause $\phi'(t) > \phi'_r(t)$ for some $t \in <t_1,t_2>$.

Mortensen (1986) gives the signs of the derivatives of the reservation wage with respect to exogenous variables in a stationary model. Those results are in accordance with Theorem 3 (take the reference model and the alternative model to be stationary, so $t_1=0$, $t_2=\infty$).

3.3.4. *The unemployment duration distribution*

Given the results concerning the time path of the reservation wage, we can construct the unemployment duration distribution in a nonstationary job search model and extend the comparative dynamics analysis using this distribution. Define the hazard $\theta(t)$ of leaving unemployment at time t as

(7) $\theta(t) = \lambda(t).\bar{F}(\phi(t);t)$

By virtue of Assumption 1, $\theta(t)$ is a continuous function of $\phi(t)$. From Theorem 1 and Assumption 3 then, $\theta(t)$ is a continuous function of t except for points of time at which $\lambda(t)$ or $F(w;t)$ are discontinuous functions of t.

The unemployment duration density is given by the well-known equation

$$(8) \qquad h(t) = \theta(t).\exp\{ -\int_0^t \theta(u)du \}$$

From the continuity of $\phi(t)$ and the piece wise continuity of $\overline{F}$ and λ as a function of t and from the boundedness of $\overline{F}$ and λ, it is clear that the integral in equation (8) exists for every $t \geq 0$. For points of time at which $\theta(t)$ is a continuous function of t, $h(t)$ is continuous as well, and vice versa. Note that though $h(t)$ is discontinuous at points where $\lambda(t)$ or $F(w;t)$ are discontinuous functions of t, the distribution function associated with $h(t)$ is a continuous function of t on the whole interval $[0,\infty\rangle$.

The expected duration of unemployment can be written as

$$(9) \qquad E(t) = \int_0^\infty \exp\{ -\int_0^t \theta(u)du \}dt$$

Note that this expression may not exist. E.g. if for every $t \geq 0$ $\lambda(t)=0$ then also $\theta(t)=0$ and people remain unemployed forever. Sufficient for existence is that $\lambda(T)>0$ and $b(T)<\beta(T)$ for then $\theta(T)>0$ and $E(t) \leq T + (1/\theta(T))$. From (7) we infer that if for some t $\alpha(t)<\phi(t)<\beta(t)$ and $\lambda(t)>0$ then shifts in benefits that cause a rise of $\phi(t)$ also cause a fall of $\theta(t)$. Because of the continuity to the right of $\phi(t)$, $\lambda(t)$ and $F(w;t)$ as a function of t, $\theta(t)$ will fall in a neighbourhood of t. Consequently, we have as a corollary from Theorem 3,

Corollary.

Let Assumptions 1–5 *be satisfied. If* $b(t)$ *is raised for every* $t\in[t_1,t_2\rangle$ *with* $0\leq t_1<t_2\leq\infty$ *such that the new* $b(t)$ *also satisfies Assumptions* 2–4, *and if there is a point* t_3 *with* $0\leq t_3<t_2$ *at which* $\alpha(t_3) < \phi(t_3) < \beta(t_3)$ *and* $\lambda(t_3) > 0$ *then the expected duration of unemployment increases if it exists.*

In the appendix sufficient conditions for $\alpha(t_3) < \phi(t_3) < \beta(t_3)$ are given.

3.4. Exogenous variables as step functions of time

In the sequel we adopt an additional assumption, namely:

6. F(w;t), $\lambda(t)$ *and* b(t) *are step functions of* t *on* $[0,\infty>$.

For b(t) in particular this is what is often seen in practice.

We can now split the positive real axis on which time is measured into intervals within which the exogenous variables are constant. On such an interval, equation (2) reduces to a constant coefficient differential equation. Moreover, this differential equation has a stationary solution (i.e. the solution for which $\phi'(t)=0$) which is constant on that interval. This solution corresponds to ϕ_0 as it is defined by equation (4) in a more general setting. In Subsection 3.3.2 it was shown that $\phi_R'(t)$ can be considered to be a monotonically increasing function of $\phi(t)$. This also holds for $\phi_L'(t)$. Further, if, in a model that satisfies Assumptions 1–6, $\phi'(t)$ exists for some t, then so does $\phi''(t)$. By differentiating the constant coefficient differential equation with respect to t we find that $\phi'(t)$ and $\phi''(t)$ have equal sign. Thus we have the following information about the shape of $\phi(t)$ within intervals on which the exogenous variables are constant:

Theorem 4.

Let Assumptions 1–6 *be satisfied. Let the exogenous variables be constant on an interval* $[t_*,t^*>$, $0 \le t_* < t^* \le \infty$. *Then for every* $t \in <t_*,t^*>$

$$\phi(t) \lesseqgtr \phi_0 \;\Leftrightarrow\; \phi'(t) \lesseqgtr 0 \;\Leftrightarrow\; \phi''(t) \lesseqgtr 0 \Leftrightarrow$$

$$\phi(t_*) \lesseqgtr \phi_0 \;\Leftrightarrow\; \phi_R'(t_*) \lesseqgtr 0 \;\Leftrightarrow\; \phi(t^*) \lesseqgtr \phi_0 \Leftrightarrow\; \phi_L'(t^*) \lesseqgtr 0.$$

Deviations of $\phi(t)$ from ϕ_0 arise because of anticipations of future changes of the values of exogenous variables. As time proceeds, these changes come nearer. Now the rate of discount is positive and the probability of finding a job before the end of the present interval when following the optimal strategy decreases when t rises. Therefore anticipations become stronger and ϕ shifts away further from ϕ_0. As ϕ is the only variable that changes within the interval, this in turn implies that ϕ' increases in absolute value, which explains the sign of ϕ''. Note that the sign of $\phi - \phi_0$ at the end point of an interval can be thought of as determining the sign of the slope of ϕ within the interval.

Now suppose there is only one point in time T at which exogenous variables are allowed to change values. In addition, suppose that only one exogenous variable changes in value at T, according to one of the following four rules: (if necessary, values of the exogenous variables before and after T will be distinguished by subscripts 1 and 2 respectively)

D_1) $b_1 > b_2$.

D_2) $\lambda_1 > \lambda_2$ *while* $\phi(T) < \beta$.

D_3) F_1 *first-order stochastically dominates* F_2 *while* $\phi(T) < \beta_1$ *and* $\lambda > 0$.

D_4) F_1 *is a mean-preserving spread of* F_2 *while* $\alpha_1 < \phi(T) < \beta_1$ *and* $\lambda > 0$.

Let ϕ, and ϕ_2 denote the stationary solutions on the time intervals $[0,T>$ and $[T,\infty>$, respectively. Whether $\phi_1 \lesseqgtr \phi_2$ follows from the well-known comparative statics results in a stationary model. E.g. in a stationary model an increase in b implies an increase in the stationary reservation wage, so in case D_1 $\phi_1 > \phi_2$ and consequently $\phi(T) < \phi_1$. Theorem 4 can then be applied in order to obtain the following

<u>Corollary.</u>
Let Assumptions 1–6 *be satisfied. Let* T *be the only one point in time at which exogenous variables are allowed to change values, according to* D_1, D_2, D_3 *or* D_4. *Then* $\phi_1 > \phi_2$ *and*

(i) *for every* $t \in [0,T>$, $\phi_2 < \phi(t) < \phi_1$, $\phi'(t) < 0$, $\phi''(t) < 0$.

(ii) $\phi(T) = \phi_2$, $\phi'_L(T) < 0$, $\phi'_R(T) = 0$.

Again, simultaneous occurrence of some D_1, D_2, D_3, D_4 can be examined by sequential application of the corollary. Note that a part of this corollary can also be proved using Theorem 3. Burdett (1979) and Mortensen (1977) proved that in case D_1, $\phi'(t)<0$ for every $t \in [0,T>$, in a model in which time devoted to search is endogenous. Mortensen (1986) also proved that for every $t \in [0,T>$ $\phi'(t)<0$ if, in terms of our model, both λ and b decrease at T.

The results in Sections 3.3 and 3.4 hold for the basic job search model as outlined in Section 3.2. However, likewise results can be obtained for models that are more realistic in some respects. For instance, in reality one generally knows the wage rate associated with a vacancy before one responds to that vacancy, i.e. before the job is actually offered. Narendranathan & Nickell (1985) constructed a search model that deals with this. In Chapter 2 it is shown that such a model can be rewritten as the model described in Section 3.2 though of course the interpretation of some of the variables changes. Some of the papers in which stationary structural search models are estimated assume utility maximization instead of income maximization (Narendranathan & Nickell (1985), van den Berg (1990c)), so it may be worthwhile to examine in what sense the results are affected if utility is nonlinear. Assume that utility is intertemporally separable, the instantaneous utility function being u(w) in case one works at a wage rate w and v.u(b(t)) in case one is unemployed for t periods, receiving benefits b(t). The parameter v represents the non-pecuniary component of instantaneous utility in unemployment relative to employment. In order to obtain elegant results and in order to restrict attention to economically meaningful cases, it is assumed that u is a differentiable function on $<0,\infty>$ with for every $t\in<0,\infty>$ $u'(t)>0$ and that $E_{w;t}(u(w))$ is a uniformly bounded function of t. Further, v has to be positive. If u(0) is not defined then b(t) has to be positive for every $t\geq0$. It can be proved that Theorem 1 holds in such a model with $\phi(t)$ satisfying

$$(10) \qquad u'(\phi(t)).\phi'(t) = \rho.u(\phi(t)) - \rho.v.u(b(t)) - \lambda(t).\int_{\phi(t)}^{\infty} u'(w).\overline{F}(w;t)dw$$

in all points at which $\phi'(t)$ is defined. Again, (10) can be used to calculate the right-hand derivative and left-hand derivative in points at which $\phi'(t)$ is not defined. The model can be rewritten in terms of the basic model, defining e.g. a transformed level of benefits $b^*(t)$ as v.u(b(t)). After doing so the other theorems can be applied to obtain results for the model with utility maximization.

3.5. An empirical illustration

3.5.1. *Introduction*

In this section we present the results of the estimation of a nonstationary structural job search model in order to illustrate the importance of allowing for nonstationarity. One of the main items in the applied literature on unemployment duration is the magnitude of the effect of a change in the benefits level on the expected duration (for a survey, see e.g. Atkinson (1988)). However, though generally it is acknowledged that in most countries benefits are a decreasing function of duration, the models used in empirical analyses do not deal with this (for references, see Section 3.1). The structural models that are used erroneously assume that b is constant throughout duration. Estimated reduced-form models of duration generally allow for (parametric) duration dependence that acts multiplicatively on the hazard θ whereas the observed b is treated as a constant (i.e. non-time-varying) regressor in θ. Moffitt (1985) argued that such a proportional hazard model cannot be a satisfactory representation of the duration dependence due to decreasing benefits since this acts in a non-proportional way on the hazard. This is an important point because, as will be shown, these decreases can be substantial. It is clear that in a structural nonstationary setting such problems do not exist. Nickell (1979) and Narendranathan, Nickell & Stern (1985) estimate proportional hazard models in which both b and the coefficient in the hazard associated with b are allowed to vary across different periods. Though the models are more general than the proportional hazard models that are commonly used, they are not able to represent some of the essential features of nonstationarity due to decreasing benefits. First, and most important, the models do not allow for anticipation of future changes of the level of benefits. The specified hazard θ at t depends on the present level of benefits b(t) only and is not allowed to depend on future values of b which in fact may have a large influence on the present reservation wage and therefore also on $\theta(t)$. Another objection to these models is that no account is taken of the diminishing influence of the level of benefits within a period, as time proceeds towards the end of that period.

Using micro data from 1983 on unemployed individuals we estimate a nonstationarity structural model that allows for decreasing benefits. In The Netherlands in the beginning of the eighties the benefits level during the first years of unemployment is related to the pre-unemployment wage while after that it is determined by the public assistance system. As a consequence, for an individual who has had a job before becoming unemployed benefits generally decrease substantially when duration equals about 2 years. Such a decrease does not occur if the benefits level related to the pre-unemployment wage is below the public assistance level, of if the individual did not have a job before becoming unemployed e.g. because he is a new entrant on the labour market. In those cases he obtains public assistance benefits from the beginning. The data used to estimate the model are obtained from a survey among some 400 males in Amsterdam. The sampling scheme of the survey was meant to over represent unemployed individuals, but it makes no reference to the benefits paths of the respondents or to factors that determine those paths. As a result the level of benefits at 2 years of unemployment does not decrease for all unemployed individuals in the sample. Though the sample is somewhat small, it contains some interesting information on the labour market environment and the behaviour of the respondents including subjective responses on reservation wages. This information is extensively used in the analysis. From the estimated model we can calculate sample averages of the elasticities of the expected duration with respect to the levels of benefits before and after 2 years of unemployment. Information on the magnitudes of such elasticities may be valuable for policy makers.

3.5.2. *The model*

We use the search model described at the end of Section 3.4. Analogous to Narendranathan & Nickell (1985) and van den Berg (1990c) the utility function of income u is logarithmic so we assume that individuals are risk averse. For the wage offer density f(w) the following functional form is chosen

$$
(11)\qquad
\begin{aligned}
f(w) &= \frac{1}{w}\,\frac{1}{\log \beta/\alpha} && \text{if } \alpha \le w \le \beta \\
f(w) &= 0 && \text{elsewhere}
\end{aligned}
$$

with $0<\alpha<\beta<\infty$. This distribution is positively skewed and rules out wage offers close to zero or infinity. Moreover, in the stationary version of the model with wage offer density (11) it always holds that $\partial\theta/\partial\lambda\geq0$ (This inequality does not follow for every conceivable class of wage offer distributions). A particular advantage of specification (11) is that we can use subjective responses on α and β to estimate individual wage offer distributions.

Nonstationarity arises if the level of benefits decreases when unemployment duration t equals T months. After t=T the reservation wage is constant and can be calculated by imposing $\phi'(t)=0$ in equation (10). Before t=T $\phi(t)$ follows the differential equation (10). For the functional forms of u and F(w) mentioned above this is a first-order nonlinear differential equation in log $\phi(t)$ with constant coefficients. It can be solved using the boundary condition $\phi(T)$.

3.5.3. *The data and the empirical implementation of the model*

The data were obtained from a survey among some 400 males in Amsterdam who were at the date of the interview between 30 and 55 years old. A descriptive analysis of these data can be found in Ridder (1987). The respondents were asked to reconstruct their labour market histories over the past 10 years until the date of the interview which was between October 20 and December 18, 1983. In addition they were asked to provide information on income variables and personal characteristics. The respondents were drawn from three different sampling schemes. In all cases males in Amsterdam aged between 30 and 55 were sampled. The first subsample (RS) is a random sample from this group of individuals. From this we selected 22 individuals who were unemployed at the date of the interview. The second subsample (SS) is a sample of individuals who were unemployed at September 1, 1983. From this we selected 137 individuals. The third subsample (FS) is a sample of the inflow into unemployment around September 1, 1983. From this we selected 41 individuals, which gives a total number of 200 individuals. For RS we determined t_b, the elapsed duration of unemployment at the date of the interview. For SS we determined t_b, the elapsed duration of unemployment at September 1, 1983, and t_f, the duration of unemployment after that date. Finally, for FS we determined t_c, the duration of the spell of unemployment starting around September 1, 1983. All t_f and most t_c are censored. Because of the lack of information on income variables for past spells of unemployment, such spells could not be used. The individual log-likelihood contribution of t_c for FS is

$$(12) \qquad \mathcal{L}_{FS}(t_c) = (1-c_1)\log(\theta(t_c)) - \int_0^{t_c} \theta(t)dt$$

in which c_1=1 if t_c is censored and 0 elsewhere. Assume that the individual entry rate into unemployment is constant before the moment of the interview. Then for RS,

$$(13) \qquad \mathcal{L}_{RS}(t_b) = -\int_0^{t_b} \theta(t)dt - \log E(t)$$

while for SS

$$(14) \qquad \mathcal{L}_{SS}(t_b,t_f) = -\int_0^{(t_b+t_f)} \theta(t)dt - \log E(t)$$

(see e.g. Ridder (1984)). Recall that t_f in (14) is censored. E(t) in (13) and (14) follows from equation (9).

Individuals who were unemployed at the date of the interview were asked for their lowest acceptable net wage in a job at that date. These 'observed' reservation wages $\tilde{\phi}(t)$ may differ from the true reservation wages,

$$(15) \qquad \tilde{\phi}(t) = \phi(t) + \varepsilon$$

ε is an error term which is interpreted as a measurement error that is i.i.d.

across individuals and independent of duration t. Consequently, individuals use $\phi(t)$ instead of $\tilde{\phi}(t)$ as their strategy at t so $\theta(t)$ depends on $\phi(t)$ instead of $\tilde{\phi}(t)$ and equations (12), (13) and (14) do not depend on ε. Further, by assuming that ε has a normal distribution with mean zero and variance σ^2 we have that, conditional upon the elapsed duration t (t_c in case of FS, t_b in case of RS, t_b+t_f in case of SS), $\tilde{\phi}(t)$ has a normal distribution with mean $\phi(t)$ and variance σ^2. The total log-likelihood contribution of an individual can be written as the sum of the marginal contribution of the duration variables (see equations (12)-(14)) and the conditional contribution of the observed reservation wage. The latter equals

$$(16) \qquad (1-c_2)\left\{-\tfrac{1}{2}\log 2\pi - \log\sigma - \tfrac{1}{2}\cdot\left[\frac{\tilde{\phi}(t)-\phi(t)}{\sigma}\right]^2\right\}$$

in which $c_2=1$ if $\tilde{\phi}(t)$ is missing (16 individuals) and 0 elsewhere.

In order to be able to estimate the model additional information is required concerning F(w) (see Flinn & Heckman (1982)). It seems natural to use post-unemployment wages because these are random drawings from F(w) truncated at $\phi(t)$. However, our sample is basically retrospective and only in FS a few post-unemployment wages are observed. Moreover, F(w) as specified is not recoverable from the truncated F(w). Therefore we take a totally different route in estimating F(w). Analogous to Lancaster & Chesher (1983) and Lynch (1983) we use subjective responses on characteristics of F(w). Unemployed respondents were asked what the minimal and maximal wages were of those employed in their occupation. The questions make no references to the strategy actually used to locate potential wage offers, so the answers can be interpreted as 'observed' minimal and maximal wage offers $\tilde{\alpha}$ and $\tilde{\beta}$, respectively. Again we postulate that the true α and β are imperfectly observed by $\tilde{\alpha}$ and $\tilde{\beta}$ because of non-systematic measurement errors. We ran an OLS regression of log $\tilde{\alpha}$ and log $\tilde{\beta}$ on observed personal characteristics, using data from individuals who responded on $\tilde{\alpha}$ and $\tilde{\beta}$ (134 and 128 observations, respectively). For all 200 individuals α and β can be predicted using these estimated relationships. Analogous to Narendranathan & Nickell (1985) and van den Berg (1990c) the predicted F(w) are plugged in when estimating the structural model.

In order to estimate the model the whole benefits path b(t), $0 \le t < \infty$ has to be known rather than just the level of benefits at the moment of interview. If an individual has had a job before becoming unemployed then during the first half year of unemployment his benefits level equals 80% of the previous wage while during the next 1.5 to 2 years it equals about 70% of the previous wage. After about 2 years of unemployment he obtains public assistance benefits depending on household composition and financial characteristics of other household members. (The exact unemployment duration at which b decreases from 70% of the previous wage to the public assistance level depends on the individual's labour market history, but is generally close to 2 years. In order to keep the exposition simple, we take T to be equal to 2 years for every individual in the sample. Sensitivity checks show that the results are robust with respect to small changes in T). The decrease from 80% to 70% is not very substantial and is generally much smaller than the decrease at 2 years of unemployment, so in order not to complicate the empirical analysis we will concentrate on the latter decrease and assume that during the first two years 70% of the previous wage is obtained. If this 70% is below the public assistance benefits level then the individual obtains the latter and the model reduces to a stationary model. If the individual did not have a job before becoming unemployed (e.g. because he is a new entrant on the labour market) then he obtains public assistance benefits from the beginning and the model is stationary. As a result, for 136 of the 200 individuals the model is nonstationary. Using survey information on the (inflation-corrected) previous wage and on the level of benefits at the date of the interview and applying the rules of the public assistance system in 1983 in The Netherlands the variables b(0) and b(T) were constructed.

The job offer arrival rate λ and the relative disutility of being unemployed v are parameterized as exponential functions of observable characteristics x_1 and x_2, respectively,

$$\lambda = \exp(x_1'\beta_1) \qquad\qquad v = \exp(x_2'\beta_2)$$

The vector x_1 contains possible indicators of λ e.g. because they give an indication of the productivity of the searcher. Note that the sample is homogeneous with respect to sex and geographic area and fairly homogeneous with respect to age so these are not included in x_1 and x_2. The vector x_2 contains possible indicators of v e.g. because they give an indication of the attitude towards work of people in the direct environment of the searcher.

The estimation we have employed was ML using the BHHH algorithm. Because we can solve analytically for $\phi(t)$, $\int_0^t \theta(u)du$ and E(t) as functions of t, b(t) ($0 \leq t < \infty$), F(w), λ, v, ρ and σ^2 it follows that the likelihood can be written analytically as a (very complicated) function of the unknown parameters β_1, β_2, ρ and σ^2.

3.5.4. *The results*

The parameter estimates for the nonstationary structural model described in Subsections 3.5.2 and 3.5.3 are presented in Table 1. The unit time period is one month. For education the reference category is level 1.

Table 1. Parameter estimates for the search model.

variable/parameter	coefficient	(t–ratio)
(i) *job offer arrival rate*		
constant	–3.72	(18.5)
Dutch	0.18	(1.0)
education: level 2	0.18	(0.8)
education: level 3	0.42	(1.9)
married	0.48	(2.6)
partner has paid job	0.22	(0.9)
(ii) *disutility of unemployment*		
constant	–0.01	(0.5)
education: level 2	–0.02	(0.6)
education: level 3	–0.12	(1.9)
partner has paid job	0.01	(0.3)
(iii) *subjective rate of discount*		
(*in percent per year*)	12%	(3.4)
(iv) *standard deviation of the measurement*		
error of the reservation wage	469	(29.9)
Log–likelihood = –2095.65		

Generally, the results seem to be in accordance with intuition. Because this is merely an empirical illustration and because our main interest is in the elasticities of duration with respect to benefits we will not give a lengthy account of these results. Also, we are not particularly interested in search characteristics of unemployed individuals whose environment is stationary so the results below are only for individuals whose environment does change. Given the parameter estimates, the main variables of the search process can be estimated. Table 2 presents sample averages of the estimates of

λ, $\overline{F}(\phi(0))$ and $\overline{F}(\phi(T))$ for different levels of education. The acceptance probability increases by about 0.1 from the moment that one becomes unemployed until the moment that one is unemployed for 2 years. After 2 years of unemployment most job offers are acceptable. Rejection of an offer may well imply a waiting time of more than a year before the next offer arrives. (This result is also found in other studies on unemployment in The Netherlands in the beginning of the eighties, see Chapter 2.) In the meantime the only source of income is public assistance benefit which generally is much smaller than α. Moreover, because $v<1$ one also dislikes being unemployed for non-pecuniary reasons.

Table 2. Probabilities and expectations.

level of education	1	2	3
λ (expected number of offers)	0.040	0.047	0.060
$\overline{F}(\phi(0))$(proportion of offers acceptable at t=0)	0.78	0.68	0.87
$\overline{F}(\phi(T))$(proportion of offers acceptable after 2 years)	0.88	0.83	0.95

The results so far enable us to investigate a number of questions related to the effectiveness of policies aimed at a reduction of unemployment durations. Table 3 presents for different levels of education, sample averages of the elasticities of the reservation wages $\phi(0)$ and $\phi(T)$ and the expected duration $E(t)$ with respect to the levels of benefits $b(0)$ and $b(T)$. The effects of a simultaneous proportional change of $b(0)$ and $b(T)$ are found by summing the elasticities in part (i) and part (ii) of Table 3.

Of course the elasticity of $\phi(T)$ with respect to $b(0)$ is identically zero: the optimal strategy does not depend on past income. What strikes is that all other elasticities for the highest level of education are smaller than the corresponding elasticities for levels 1 and 2. Highly educated individuals dislike being unemployed for non-pecuniary reasons more than others do. Further, the job offer arrival rate and the difference between the mean wage offer and the level of benefits are larger for them. Consequently, the expected duration is much shorter and the expected present value of search is not dominated by the prospect of being dependent on benefits for a long time.

Table 3. Elasticities with respect to benefits.

level of education	1	2	3
(i) *with respect to the level of benefits before 2 years*			
$\frac{\partial \log \phi(0)}{\partial \log b(0)}$ (reservation wage at t=0)	0.15	0.14	0.11
$\frac{\partial \log \phi(T)}{\partial \log b(0)}$ (id. after 2 years)	0	0	0
$\frac{\partial \log E(t)}{\partial \log b(0)}$ (expected duration)	0.14	0.16	0.07
(ii) *with respect to the level of benefits after 2 years*			
$\frac{\partial \log \phi(0)}{\partial \log b(T)}$	0.09	0.08	0.04
$\frac{\partial \log \phi(T)}{\partial \log b(T)}$	0.23	0.21	0.14
$\frac{\partial \log E(t)}{\partial \log b(T)}$	0.47	0.59	0.06

For levels of education 1 and 2 the most striking feature of Table 3 is that the elasticity of the expected duration with respect to the benefits level after 2 years of unemployment (the public assistance benefits level) is much larger than the corresponding elasticity with respect to the level before 2 years (the pre-unemployment-wage-related benefits level). This implies that a decrease of b(T) would be much more effective in reducing durations than a decrease of b(0) would be. Note that changing the value of b(T) affects the reservation wage $\phi(t)$ on the whole time interval $[0,\infty>$ whereas changing the value of b(0) only affects $\phi(t)$ on $[0,T>$. Moreover, the influence of b(0) on $\phi(t)$ is diminishing at t proceeds on $[0,T>$. For levels of education 1 and 2 the anticipation on $t<T$ of the decrease of the benefits level at T is quite strong because the probability of getting a job during the first 2 years of unemployment is rather small. In other words, the short-term unemployed individuals' strategy is sensitive with respect to changes of the benefits level for the long-term unemployed because they know they may well become

long–term unemployed themselves.

Information on the magnitudes of such elasticities may be valuable for policy makers. E.g. shifting b(t) on $t \geq T$ is almost as effective in reducing duration of individuals with level of education 1 or 2, as shifting the whole benefits path. The estimated model can be used for simulating alternative benefits policies. For every alternative benefits path the optimal strategy can be solved from equation (10). Note that all these results can not be obtained by using stationary models.

The empirical model used in this section may be restrictive in some respects. For instance, it was assumed that λ and F(w) are stationary. Moreover, we did not allow for transitions into a third state, say nonparticipation. These features can be implemented but the empirical analysis of such extended models requires more data and is a task for further research.

When deriving the likelihood no account has been taken of unobserved heterogeneity in the sample, which may bias the results. However, from a numerical point of view the inclusion of a random heterogeneity term would complicate things enormously even in a stationary model, so it would be beyond the scope of this illustration to do so.

3.6. Conclusion

In this chapter we have examined nonstationarity in job search theory. The optimal reservation wage path over time has been derived under weak assumptions concerning the exogenous variables. We also have given comparative dynamics results. Furthermore, by assuming the exogenous variables to be step functions of time we were able to derive additional properties of the reservation wage path. Generally these properties are in accordance with economic intuition. As an empirical illustration we estimated a nonstationary structural job search model. The model allows for the level of benefits to be a decreasing function of unemployment duration, which is a stylized fact in most countries. It appeared that generally the elasticity of the expected duration with respect to the level of benefits after two years of unemployment is much larger than the elasticity with respect to the level of benefits that is obtained in the first two years of unemployment.

There are some straightforward directions for further research. Instead of assuming that unemployed individuals have perfect foresight with respect to the future time paths of b, λ and F(w), it might be more realistic to allow for stochastic changes in these objects. These may be due to such things as unforeseen changes in aggregate macroeconomic conditions or changes in personal circumstances. It then seems reasonable to assume that individuals are aware of these additional elements of uncertainty and derive their optimal strategies given some (subjective) assessment of the probabilities that such changes occur. The analysis of the optimal strategy is much more complicated in such nonstationary models because $\phi(t)$, if it exists, is not only a function of time but also of the realizations at t of the stochastic elements in b, λ and $F(w)$. Also, the empirical analysis will be much harder because the probability assessments of the changes generally appear explicitly in the structural model.

At the end of the empirical illustration in Section 3.5 we mentioned some apparent rigidities of the model specification used. A task for future empirical research is to relax those rigidities.

Appendix to Chapter 3

3.A.1. Proof of Theorem 1

For a derivation of the properties of the optimal strategy it is necessary to examine in detail the expected present value of income when unemployed. Individuals who are unemployed for t units of time are assumed to maximize the following expression

$$\text{(A1)} \qquad E \int_t^\infty e^{-\rho(\tau-t)}\, y(\tau)d\tau$$

in which $y(\tau)$ denotes the income flow at τ and expectation is taken over job offer arrival times and wage offers. Let R(t) denote the expected present value of income at t when following the optimal strategy. Then R(t) is the supremum of expression (A1) over all admissible policies. For nonstationary decision processes a recursive (Bellman's) equation in terms of the optimal value generally does not follow trivially from some optimality principle. Indeed the derivation of such an equation would need a rather heavy measure-theoretic apparatus (see e.g. Hinderer (1970)) and the optimal control literature on such problems in a continuous time nonstationary context is not very well developed yet (see Whittle (1983)). Therefore such a task is beyond the scope of the chapter and a different route is followed: the recursive relation is stated and it is proved that there exists a unique solution R(t) which is bounded and continuous in t and which can be differentiated with respect to t almost everywhere. Using the relation between R(t) and the reservation wage, the desired properties of the latter can be deduced.

R(t) is written recursively as a function of $R(\tau)$, $\tau>t$, in which τ is interpreted as the point of time at which the next offer arrives (so $\tau-t$ is the waiting time until the next offer). First the distribution of τ given t has to be derived. The job offer probability in a small interval $[\tau,\tau+d\tau>$ conditional on not having received an offer between t and τ and conditional on being unemployed at t, is $\lambda(\tau)d\tau$. Defining $G(\tau;t)$ to be the distribution function of τ for someone whose elapsed duration equals t, we have the familiar result

$$\text{(A2)} \qquad G(\tau;t) = 1 - \exp\left\{ -\int_t^\tau \lambda(s)ds \right\} \qquad \tau\geq t$$

Because λ is uniformly bounded and continuous almost everywhere in t, equation

(A2) properly defines $G(\tau;t)$ for every $t \geq 0$. Note that Assumption 2 allows for $\lambda(t)=0$ for every t. In such a case the state of unemployment is absorbing because job offers never arrive: $G(\tau;t) = 0$ for every τ so τ has a defective distribution.

Now $R(t)$, $0 \leq t < \infty$, can be written recursively as

$$\text{(A3)} \qquad R(t) = \int_t^\infty \left[\int_t^\tau b(s)e^{-\rho(s-t)}\, ds + e^{-\rho(\tau-t)}.E_{w;\tau}\max\left(\frac{w}{\rho}, R(\tau)\right) \right] dG(\tau;t)$$

From t to τ the individual receives benefits; at τ he has to choose between acceptance of a job offer (present value w/ρ) and rejection of it (present value $R(\tau)$). From Assumption 4 it follows that if duration t exceeds T then the model breaks down to a stationary model. Therefore $R(t)$ is constant for $t \geq T$ and, as has been shown often before, $R(T)$ is the unique finite solution to

$$\text{(A4)} \qquad \rho R(T) = b(T) + \frac{\lambda(T)}{\rho} E_{w;T} \max\left(\frac{w}{\rho} - R(T), 0\right)$$

if the assumptions on boundedness hold. Consequently, further analysis of equation (A3) can be restricted to $t \in [0,T]$.

It is rather straightforward but tedious to show that if the assumptions on semi-continuity and boundedness hold, then the right-hand side of equation (A3) is a mapping $M(R)$ which maps the space of continuous functions on $[0,T]$ into itself, and the integrals in (A3) are well-defined (see e.g. Haaser & Sullivan (1971)). Let $C[0,T]$ denote the space of continuous functions on $[0,T]$, normed with the sup-norm. Then $C[0,T]$ is a Banach space. We now show that M is a contraction mapping, i.e. that there is an $\alpha \in \langle 0,1 \rangle$ such that for every $R,R^* \in C[0,T]$ it holds that $\|M(R) - M(R^*)\| \leq \alpha.\|R - R^*\|$. We have

$$\text{(A5)} \qquad \|M(R) - M(R^*)\| = \sup_{0 \leq t \leq T} |M(R)(t) - M(R^*)(t)| =$$

$$\sup_{0 \leq t \leq T} \left| \int_t^\infty e^{-\rho(\tau-t)}.E_{w;\tau}\left(\max\left(\frac{w}{\rho}, R(\tau)\right) - \max\left(\frac{w}{\rho}, R^*(\tau)\right) \right) dG(\tau;t) \right|$$

$$\leq \sup_{0 \leq t \leq T} \int_t^\infty e^{-\rho(\tau-t)}.E_{w;\tau} \left| \max\left(\frac{w}{\rho}, R(\tau)\right) - \max\left(\frac{w}{\rho}, R^*(\tau)\right) \right| dG(\tau;t)$$

Because for every $x,y,z \in \mathbb{R}$, $|\max(x,y)-\max(x,z)| \leq |y-z|$, the expression above is bounded by

$$\sup_{0\leq t\leq T} \int_t^\infty e^{-\rho(\tau-t)}.|R(\tau) - R^*(\tau)|\ dG(\tau;t)$$

$$\leq \sup_{0\leq t\leq T} \int_t^\infty e^{-\rho(\tau-t)}.\left[\sup_{0\leq\tau\leq T} |R(\tau) - R^*(\tau)|\right]\ dG(\tau;t)$$

$$= \sup_{0\leq t\leq T} |R(t) - R^*(t)|\ .\ \sup_{0\leq t\leq T} \int_t^\infty e^{-\rho(\tau-t)}\ dG(\tau;t)$$

The second part of the right–hand side of the last equation does not depend on R or R^* so it is now sufficient to prove that

$$\text{(A6)} \qquad \sup_{0\leq t\leq T} \int_t^\infty e^{-\rho(\tau-t)}\ dG(\tau;t) \in [0,1\rangle$$

One sees immediately that the supremum lies in the interval [0,1]. It remains to prove that 1 is never obtained. If, $\forall t\geq 0$, $\lambda(t)=0$, then expression (A6) equals zero. If there is a $t\geq 0$ with $\lambda(t)>0$ then, from the semi–continuity of λ as a function of time and from the positiveness of ρ, it follows that the expression is strictly bounded from above by the supremum over $0\leq t\leq T$ of $1-G(t;t)$, which never exceeds 1. Consequently, (A6) holds and M is a contraction mapping. From Banach's theorem (Wouk (1979)) it follows that M which is defined on C[0,T] has a unique fixed point. So, from equation (A3), a function R(t) in C[0,T] exists and is the unique continuous function that solves equation (A3). Because of the stationarity after T, the latter can be extended to R(t) on $[0,\infty\rangle$.

Equation (A3) can be used to derive the derivative of R(t) with respect to t. It follows that

$$\text{(A7)} \qquad R'(t) = \rho R(t) - b(t) - \frac{\lambda(t)}{\rho} \int_{\rho R(t)}^\infty (w - \rho R(t))\ dF(w;t)$$

in which the integral can be simplified to

$$\int_{\rho R(t)}^\infty \overline{F}(w;t)\ dw$$

by partial integration. Differentiation is only allowed if λ, b and F(w) are continuous functions of time at t. However, because these functions are always continuous from the right, the right–hand side of (A7) gives the right–hand derivative of R(t) with respect to t at points at which the exogenous variables are discontinuous. Similarly, because the left-hand limits of these variables exist, the left-hand side of R(t) with respect to t at such discontinuity points is defined by

$$\text{(A8)} \qquad R'_L(t) = \rho.R(t) - \lim_{\tau \uparrow t} b(\tau) - \lim_{\tau \uparrow t} \lambda(\tau) \, .\lim_{\tau \uparrow t} \int_{\rho R(t)}^{\infty} \overline{F}(w;\tau)dw$$

It is clear from equation (A3) that the optimal policy is to accept a wage offer w at time t if and only if w/ρ exceeds R(t). In other words, the present value of working at a wage w has to exceed the present value of searching further in the optimal way. Consequently the optimal policy can be characterized by a reservation wage $\phi(t)$ defined by

$$\text{(A9)} \qquad \phi(t) = \rho.R(t)$$

and the theorem follows from the results on R(t). Note that the reservation wage function determined by the unique solution to (2) characterizes the optimal policy of an individual whose aim it is to maximize the objective function (A1) under the assumption that the optimized value of (A1) satisfies the recursive (Bellman's) equation (A3).

If $\phi(t)$ does not lie between the upper and lower bound of the interval on which F(w;t) increases then there are many other reservation wages that are able to characterize the optimal strategy. Still, $\phi(t)$ as defined by (A9) can be used any time to describe optimal behaviour.

Finally, it should be noted that Theorem 1 can be proved without using Assumption 4.

3.A.2. Proof of Theorem 2

The structure of the proof is as follows. First we restrict attention to an unspecified time interval within which the exogenous variables are continuous. In Lemma A1 we show that sufficient for (i) and (ii) to hold in the interval is that, loosely speaking, $\phi_0(t)$ is strictly decreasing within that interval. The remainder of the proof is concerned with finding conditions that impose the required property to $\phi_0(t)$ for every interval, using backward induction.

We split the time axis into a finite number of intervals, within which every exogenous variable is continuous in time. The intervals are closed to the left side and open to the right. The last interval is $[T,\infty>$. Now consider one such interval, say $[t_*,t^*>$. From Theorem 1, ϕ is a differentiable function of t and ϕ_0 is a continuous function of t on $[t_*,t^*>$. Further, $\phi_L(t^*) = \phi(t^*)$ but it may be that $\phi'_L(t^*) \neq \phi'_R(t^*)$ or $\phi_{0_L}(t^*) \neq \phi_0(t^*)$.

Lemma A1.

Let Assumption 1–5 be satisfied. Consider the interval $[t_*,t^*>$ as defined before. If $\phi(t^*) \leq \phi_{0_L}(t^*)$ and if

$$\text{(A10)} \qquad \forall t\in[t_*,t^*>,\ \forall \tau\in<0,t^*-t>,\ \ \phi_0(t+\tau) < \phi_0(t)$$

then $\forall t\in[t_*,t^*>$, $\phi(t) < \phi_0(t)$, $\forall t\in<t_*,t^*>$, $\phi'(t) < 0$; $\phi'_R(t_*) < 0$ and if $\phi_{0_L}(t^*) > \phi(t^*)$ then $\phi'_L(t^*) < 0$ while if $\phi_{0_L}(t^*) = \phi(t^*)$ then $\phi'_L(t^*) = 0$.

Proof of lemma A1

Suppose that at some $t\in[t_*,t^*>$ $\phi_0(t) \leq \phi(t)$ holds. Then, from the discussions of equations (5) and (6) in Subsection 3.3.2, $\phi'(t)\geq 0$ if $t>t_*$, while $\phi'_R(t) \geq 0$ if $t=t_*$. On the other hand, $\phi(t^*) \leq \phi_{0_L}(t^*)$. ϕ and ϕ_0 are continuous functions of t on $[t_*,t^*>$ and ϕ_0 is decreasing in t. Therefore $\phi_0(t) \leq \phi(t)$ cannot hold for any $t\in[t_*,t^*>$. If $\phi_0(t) > \phi(t)$ for every $t\in[t_*,t^*>$ then, again from Subsection 3.3.2, $\phi'(t)<0$ for every $t\in<t_*,t^*>$ and $\phi'_R(t_*) < 0$. Furthermore, if $\phi_{0_L}(t^*) > \phi(t^*)$ then $\phi'_L(t^*) < 0$ while if $\phi_{0_L}(t^*) = \phi(t^*)$ then $\phi'_L(t^*) = 0$. This completes the proof of Lemma A1.

Basically, we now only have to prove that ϕ_0 is decreasing in t. Consider case K_2. For every $t\geq T$ $\phi(t) = \phi_0(t)$ holds, due to the stationarity after T. If λ is discontinuous at T, then $\lambda_L(T) > \lambda(T)$. Because $b<\beta$ holds,we have for every $t\geq 0$ that $\phi_0(t) < \beta$ holds (see equation (4)). Consequently, $Q(\phi_0(t)) > 0$ and therefore $\lambda_L(T) > \lambda(T)$ implies $\phi_{0_L}(T) > \phi_0(T)$, as can be seen from equation (4). If $\lambda_L(T) = \lambda(T)$, then $\phi_{0_L}(T) = \phi_0(T)$, So in any case $\phi_0(T) \leq \phi_{0_L}(T)$. Now consider the interval $[t_*,t^*>$ with $t^* = T$. Take a $t\in[t_*,t^*>$ and a $\tau>0$. Then, because b and F(w) are constant in case K_2,

$$\text{(A11)} \qquad \phi_0(t+\tau) - \phi_0(t) = \frac{\lambda(t+\tau)}{\rho}.\{Q(\phi_0(t+\tau) - Q(\phi_0(t))\}$$

$$+ \frac{1}{\rho}.Q(\phi_0(t)).\{\lambda(t+\tau) - \lambda(t)\}$$

Again, $Q(\phi_0(t)) > 0$. Further, $\lambda(t+\tau) < \lambda(t)$. Inspection of (A11) shows that therefore $\phi_0(t+\tau) \geq \phi_0(t)$ cannot hold. Because this is true for every $t\in[t_*,t^*>$ and for every $\tau>0$, we infer that (A10) holds for the interval ending at T.

So the conditions of Lemma A1 are satisfied and we can apply it, noting that $\phi_{0_L}(t^*) > \phi_0(t^*)$ if $\lambda_L(t^*) > \lambda(t^*)$ while $\phi_{0_L}(t^*) = \phi_0(t^*)$ if λ is continuous at t^*. In the latter case $\phi_L'(t^*) = 0$ of course implies $\phi'(T)=0$.

As for the interval $[u,t_*>$ before $[t_*,t^*>$ with $t^* = T$, we can go through the same lines of argument. We have seen that $\phi(t_*) < \phi_0(t_*)$. Again, λ may be discontinuous at t_*. In that case it follows that $\phi_{0_L}(t_*) > \phi_0(t_*)$. So $\phi(t_*) < \phi_{0_L}(t_*)$ holds in any case. Further, ϕ_0 decreases in t on $[u,t_*>$ and Lemma A1 can be applied again. Going backward in time, one thus obtains Theorem 2 for case K_2. Proofs of the other cases are analogous.

3.A.3. Proof of Theorem 3

We split the time axis into a finite number of intervals, within which all exogenous variables from both models are continuous functions of time. The intervals are closed to the left and open to the right. We let t_1 and t_3 be left-hand bounds of an interval and we let t_2 be the right-hand bound of an interval. Now consider one of the intervals, say, $[t_*,t^*>$. From Theorem 1, ϕ and ϕ_Γ are differentiable functions of t on $[t_*,t^*>$. Further, ϕ and ϕ_Γ are continuous at t_* and t^* but they may not be differentiable at those points.

We outline the proof of case C_2. Just like the proof of Theorem 2, we work backward in time. First, suppose $t_2<\infty$. For every $t\geq t_2$ $\phi(t) = \phi_\Gamma(t)$ holds, due to the equivalence of the exogenous variables of both models on $[t_2,\infty>$. Consider the interval $[u,t_2>$. (By definition $t_3\leq u$.) From equation (2), we have for every $t\in<u,t_2>$

$$(A12) \qquad \phi'(t) - \phi_\Gamma'(t) = \rho(\phi(t) - \phi_\Gamma(t)) - \lambda(t).\{Q(\phi(t);t) - Q(\phi_\Gamma(t);t)\}$$

$$+ \{\lambda_\Gamma(t) - \lambda(t)\}.Q(\phi_\Gamma(t);t)$$

If t=u, we replace $\phi'(t) - \phi_\Gamma'(t)$ by $\phi_R'(u) - \phi_{\Gamma R}'(u)$. As for every $t\in[u,t_2>$ $\phi_\Gamma(t) < \beta(t)$ holds, we have $Q(\phi_\Gamma(t);t) > 0$ on $[u,t_2>$. So if there is a $t\in<u,t_2>$ at which $\phi(t) \leq \phi_\Gamma(t)$ then it follows from (A12) that $\phi'(t) < \phi_\Gamma'(t)$. Also, if $\phi(u) \leq \phi_\Gamma(u)$ then $\phi_R'(u) < \phi_{\Gamma R}'(u)$. But $\phi(t_2) = \phi_\Gamma(t_2)$ and ϕ and ϕ_Γ are continuous functions of t. Therefore for every $t\in[u,t_2>$ $\phi(t) > \phi_\Gamma(t)$ has to hold. Further, according to Theorem 1,

$$\phi_L'(t_2) - \phi_{\Gamma L}'(t_2) = \{\lambda_{\Gamma L}(t_2) - \lambda_L(t_2)\} .Q_L(\phi_\Gamma(t_2);t_2)$$

which is nonpositive.

Now consider the interval $[y,u\rangle$. We just derived that $\phi(u) > \phi_r(u)$. Going through the same line of argument, if follows that for every $t\in[y,u\rangle$, $\phi(t) > \phi_r(t)$. Whether $t_3=u$ or $t_3\leq y$ does not matter for this result. We can proceed this way until we arrive at the interval of which t_1 is the right-hand bound, say $[v,t_1\rangle$. We now have for every $t\in\langle v,t_1\rangle$

$$\text{(A13)} \qquad \phi'(t) - \phi_r'(t) = \rho(\phi(t)-\phi_r(t)) - \lambda(t) .\{Q(\phi(t);t) - Q(\phi_r(t);t)\}$$

For $t=v$ we have to replace $\phi'(t)-\phi_r'(t)$ by $\phi_R'(v) - \phi_{rR}'(v)$. If there is a $t\in\langle v,t_1\rangle$ at which $\phi(t) \leq \phi_r(t)$ holds, then it follows from (A13) that $\phi'(t) \leq \phi_r'(t)$, regardless of $t \lesseqgtr t_3$. Similarly, $\phi(v) \leq \phi_r(v)$ implies $\phi_R'(v) \leq \phi_{rR}'(v)$. But $\phi(t_1) > \phi_r(t_1)$ and ϕ and ϕ_r are continuous functions of t. Therefore for every $t\in[v,t_1\rangle$ $\phi(t) > \phi_r(t)$ has to hold. Further,

$$\text{(A14)} \qquad \phi_L'(t_1) - \phi_{rL}'(t_1) = \rho(\phi(t_1) - \phi_r(t_1))$$

$$- \lambda_L(t_1) .\{Q_L(\phi(t_1);t_1) - Q_L(\phi_r(t_1);t_1)\}$$

which is positive. Also, from (A13) it follows that for every $t\in\langle v,t_1\rangle$ $\phi(t) > \phi_r(t)$ implies that $\phi'(t) > \phi_r'(t)$ while $\phi(v) > \phi_r(v)$ implies that $\phi_R'(v) > \phi_{rR}'(v)$. Backward induction leads to the results for $t<v$.

If $t_2=\infty$ we first examine the interval $[T,\infty\rangle$ on which the exogenous variables are constant. Because $T\geq t_3$ we have $Q(\phi_r(t);t) > 0$ on this interval. Therefore increasing λ in this interval induces an increasing reservation wage. Now we can go through the same line of argument as before concerning the intervals that lie to the left of T. This completes the proof in case C_2. Proofs of the other cases are analogous.

We now give sufficient conditions for the inequality restrictions on $\phi_r(t)$ on the interval $[t_3,t_2\rangle$. Without loss of generality we take $t_3\geq t_1$. Suppose that for every $t\geq t_3$ it holds that $b_r(t) < \beta_r(t)$, while $\beta_r(t)$ does not increase as a function of t on $[t_3,\infty\rangle$. Using Theorem 1, we can prove that as a result $\phi_r(t) < \beta_r(t)$ for every $t\in[t_3,t_2\rangle$. In case C_2 $\beta_r(t) = \beta(t)$ while in cases C_3 and C_4 $\beta_r(t) \leq \beta(t)$ on $[t_1,t_2\rangle$. Further, in all three cases $b_r(t) = b(t)$. This gives the sufficient condition for $\phi_r(t) < \beta(t)$ on $[t_3,t_2\rangle$. Analogously, we can prove that in case C_4 sufficient for $\phi_r(t) > \alpha(t)$ on $[t_3,t_2\rangle$ is, that $\alpha_r(t)$ does not decrease on $[t_3,\infty\rangle$ and that for every $t\geq t_3$

$$b(t) > \alpha_r(t) - \frac{\lambda(t)}{\rho} .\{E(w;t) - \alpha_r(t)\}$$

CHAPTER 4

A STRUCTURAL DYNAMIC ANALYSIS OF JOB TURNOVER AND THE COSTS ASSOCIATED WITH MOVING TO ANOTHER JOB

4.1. Introduction

In this chapter we analyze the labour market behaviour of employed individuals using a structural on-the-job search model. The model allows for nonzero costs associated with moving to another job. Using a data set which provides abundant information on the labour market environment of employed individuals we are able to estimate the structural parameters of interest.

During the last decades the use of job search models for the analysis of unemployment durations has become widespread. The reduced-form approach, in which only the hazard of the duration distribution is estimated, seems to be replaced gradually by a structural approach in which the search-theoretical framework is used explicitly in empirical analysis (some examples of the latter approach are Yoon (1981), Lancaster & Chesher (1983), Narendranathan & Nickell (1985), Ridder & Gorter (1986), Wolpin (1987) and van den Berg (1990b). Structural empirical inference allows one to estimate the underlying parameters of the search process, to formally test the adequacy of the theory and to make detailed policy recommendations.

Burdett (1978) was one of the first to model labour market behaviour of employed individuals in a job search context, to account for the fact that most job-to-job transitions occur without an intervening spell of unemployment. In these so-called on-the-job search models individuals search for jobs which are better than their present ones. By now there is an extensive theoretical literature on on-the-job search (see for example Hey & McKenna (1979), Holmlund (1984), Mortensen (1985), Albrecht, Holmlund & Lang (1986)). However, up to now there haven't been published any attempts to confront the on-the-job search model with empirical data. In light of the popularity of search theory as a tool for explaining job mobility it should be interesting to make such a confrontation. Moreover, empirical inference on labour market behaviour of employed individuals may help in understanding the behaviour of unemployed individuals. It is well-known that from a theoretical point of view the possibility of search on the job influences the optimal strategy of unemployed individuals (see e.g. Mortensen (1986)). Chapter 2 provides some empirical evidence: it is shown that the estimation results for a structural search model for the unemployed are very sensitive to the extent of wage increases during employment (e.g., due to search on the job).

In this chapter we estimate a structural on-the-job search model using micro data on individuals who were employed in 1985. When specifying the model we pay particular attention to factors that may reduce flexibility of the labour market. This is partly because of a growing policy interest in obstacles discouraging individuals to change jobs. Note that using a reduced-form analysis of job durations one cannot distinguish between the costs of moving to another job and other factors that influence duration.

In addition to data on job durations we will use subjective responses of working individuals on a question about the level of their reservation wage in order to estimate the model. To our knowledge this is the first use of 'reservation wage data' of employed individuals. (As for unemployed individuals, in a number of papers reservation wage data were used for empirical inference, see Lancaster & Chesher (1983), Lynch (1983), Ridder & Gorter (1986), Main & Shelly (1988), van den Berg (1990b)).

The outline of the chapter is as follows. In Section 4.2 the on-the-job search model specification is discussed. We examine in some detail the assumptions under which the optimal strategy of an employed individual has the reservation wage property. Section 4.3 contains a description of the data and the empirical implementation of the model. We develop an estimation method that identifies the parameters of interest without having to make assumptions about the class of wage offer distributions. Section 4.4 presents the main results. In addition to the parameter estimates we present sample averages of the main characteristics of the search process. Furthermore we pay special attention to the effects of changes in the level of the costs of moving to another job and of the job offer arrival rate on the reservation wage and the duration of a job. In Section 4.5 we examine the robustness of the model with respect to various sources of misspecification. Also it is discussed how the results relate to competing theories of labour market behaviour of the employed. Section 4.6 concludes.

4.2. The model

4.2.1. *On-the-job search theory and model specification*

The theory of on-the-job search tries to explain the behaviour of employed individuals who search for a better job (for a survey, see Mortensen (1986)).

In the basic version of the theory search and job turnover are costless so in principle everybody is engaged in search. Suppose an individual works at a wage w. Offers of new jobs arrive according to a Poisson process with arrival rate λ. Such job offers are random drawings (without recall) from a wage offer distribution $F(x)$. For the moment we assume that a job is characterized by its wage level and that jobs can be held forever. Every time a job offer arrives the decision has to be made whether to accept it or to reject it. Individuals aim at maximization of their expected discounted lifetime income (over an infinite horizon). They are assumed to know λ and $F(x)$.

Most papers on on-the-job search assume that the model is stationary (see e.g. Hey & McKenna (1979), Holmlund (1984), Mortensen (1985), Albrecht, Holmlund & Lang (1986), Burgess (1988)). This means that w, λ and $F(x)$ are assumed to be independent of the duration of being in the present job and independent of all events during the stay in the present job. Further, λ and $F(x)$ are not allowed to depend on w. Obviously these assumptions are not very realistic. The motivation for adopting stationarity is that in a nonstationary setting the model equations become intractable. Also, most empirical studies using structural job search models for the unemployed assume stationarity of the models for computational reasons. Therefore it seems to be a good strategy to start an empirical analysis of on-the-job search with a stationary model. If w, λ and $F(x)$ are approximately constant within jobs and if λ and $F(x)$ do not depend heavily on w then the results will hold approximately. In Section 4.5 a test for the stationarity assumption is presented.

The model does not allow for transitions into unemployment. From a conceptual point of view such an extension can be made easily. However, our main interest is in factors influencing job-to-job transitions. Inclusion of transitions into unemployment would make the model equations more complicated and would require more data than presently used to estimate the model. In Section 4.5 it is examined in what way the estimation results may be affected by the omission of possible transitions into unemployment.

It can be argued that modelling the search process in terms of job offers is not very realistic. Sometimes one knows the wage rate associated with a job opening before the job is actually offered. Narendranathan & Nickell (1985) constructed a search model in which vacancies arrive according to a Poisson process. A vacancy is characterized by a random drawing from the distribution of wages associated with the flow of vacancies, so the decision whether to apply or not is made with knowledge of the wage corresponding to the vacancy. In Chapter 2 it is shown that such a model can be rewritten as the model described in this section with a different interpretation of λ and $F(x)$. In

Section 4.3 we show that both model versions generate the same empirical specification.

The optimal strategy of an employed individual in the environment sketched above can be characterized by a very simple rule: accept a job offer if and only if its wage exceeds the wage presently earned. The transition rate from the present job to other jobs θ can be written as the product of the job offer arrival rate and the conditional probability of accepting a job offer.

$$(1) \qquad \theta = \lambda\overline{F}(w) \qquad\qquad \overline{F} = 1-F$$

One of our main interests is in factors causing inflexibility of the labour market, that is, factors that prevent employed individuals from accepting a job offer they would have accepted in the absence of those factors. It is likely that the transaction costs associated with moving to another job are among the most important of these factors. There are numerous kinds of costs associated with moving and they may add up to a considerable amount. Moving to another job usually involves moving to another town which implies that one has to search for a new house, possibly sell the old house, make costs in order to transport furniture (though this sometimes is paid by the new employer) and redecorate the new house. Moving to another job may also be costly if other members of the household have a job too; it may be that the choice is between other members giving up their job in order to move together or splitting up the household which may incur considerable psychic costs. The loss of non-transferable pension claims is a commonly recognized transaction cost that may have a large impact on labour mobility between jobs. People who have built up large claims will be reluctant to move especially when the number of years until retirement is small. Psychic costs associated with moving may also be considerable. The family has to integrate in the new social environment while the worker has to familiarize with a new working environment. Further, he may have to learn new skills during the first period in the new job. Also, a change of the educational environment may not be beneficial for children in the household. Special financial benefits (in addition to the wage) in the present job, like fringe benefits, may prevent an individual from moving to another job if a new job does not offer benefits or offers these only after having worked for a certain length of time in that job. We extend the basic on-the-job search model by introducing transaction costs c. Specifically, every time one moves from one job to another an amount of money c has to be paid (it is assumed that non-material (psychic) costs have a monetary equivalent). Some papers have been published that analyze on-the-job search models with transaction costs (Hey & McKenna (1979), Holmlund (1984), Holmlund & Lang (1985), Burgess (1988)). Hey & McKenna (1979) and Burgess (1988) give a thorough theoretical analysis of the influence of c on labour mobility, including comparative statics results.

In all papers mentioned above c does not depend on the present wage w. However, there are various reasons to assume that c in fact does depend on w. In particular, the amount of pension claims that may be lost is related to the present wage. Also, individuals who earn a high wage may have spent more money on their house and their children's education. If the costs associated with changing houses and education are correlated with the value of the old house and the money already spent on education then c will be larger for individuals who earn a high wage.

In order to maintain stationarity we assume that c as a function of w does not depend on the time spent in the present job nor on events during the stay in the present job. In combination with the infinite horizon assumption stationarity of the model implies that the employed individual's perception of the future is independent of the time spent in the present job. Consequently, the optimal strategy is constant during the present job.

Allowing c to be a non-constant function of w has important consequences for the properties of the optimal strategy of an employed individual. The qualitative comparative statics results derived for the model with constant c do not necessarily hold anymore. Indeed, the set of acceptable wage offers may not be connected. In that case the optimal strategy does not have the reservation wage property, that is, there is no number such that a job offer is acceptable if and only if its wage exceeds that number. In Subsection 4.2.2 we derive conditions on λ, F(x), c and the subjective rate of discount ρ which ensure that the optimal strategy does have the reservation wage property.

Analogous to Hey & McKenna (1979) we do not incorporate per-period search costs in the model. This is because in our opinion actual search (noticing advertisements when reading newspapers, contacting potential employers, making an expenses-paid visit to them etc.) is relatively costless for the individuals in the data set. Also, allowing for nonzero search costs would generate computational problems when estimating the model because non-zero search costs make it optimal for some individuals not to search on the job (see e.g. Burdett (1978)). As we shall see in Section 4.3 the data suggest that in some sense all employed individuals are engaged in search.

4.2.2. *The optimal strategy of employed individuals*

Let $R(w)$ denote the expected present value of income if the present wage equals w, when following the optimal strategy. Because of the stationarity assumption $R(w)$ does not depend on the elapsed duration of the present job so $R(w)$ is constant during the present job. $R(w)$ is written recursively as a function of $R(x)$, in which x is interpreted as the wage offer associated with the next job offer. The waiting time t until the next offer has an exponential distribution with parameter λ. At t the individual has to choose between acceptance of the offer (present value $R(x)-c(w)$) and rejection (present value $R(w)$). This gives

$$(2)\qquad R(w) = \int_0^\infty \left[\int_0^t we^{-\rho s}ds + e^{-\rho t}.E_x(\max(R(x)-c(w),\ R(w))) \right]\lambda e^{-\lambda t}dt$$

$$= \frac{1}{\rho+\lambda}\ .(w + \lambda.E_x(\max(R(x)-c(w),\ R(w))))$$

A wage offer x is acceptable if $R(x)-c(w)>R(w)$ while it is not if $R(x)-c(w)<R(w)$. If $R(x)-c(w)=R(w)$ then the individual is indifferent with respect to accepting the offer or not.

The following example shows that contrary to virtually all search models for labour market behaviour our model does not guarantee the reservation wage property to hold. Suppose that $c(w)$ is constant except for a large discrete upward jump at say w_0, e.g. $c(w)=0$ for $w<w_0$, $c(w)$ is 'large' for $w\geq w_0$. An individual earning $w_0-\varepsilon$ will accept a wage offer $w_0-½\varepsilon$. However, it is conceivable that he will reject an offer w_0 because the wage increase ε does not offset the increase of c that has to be paid for another transition. So in such a case the optimal strategy does not have the reservation wage property. Moreover, there always are sufficiently high wage offers that do offset the increase of c, so the set of acceptable offers is not connected. We now present some propositions regarding existence, uniqueness and properties of $R(w)$ as given in (2) and regarding the characterization of the optimal strategy, given conditions on the structural parameters λ, $F(x)$, $c(w)$ and ρ. First these conditions are listed.

1. $0<\lambda<\infty$, $0<\rho<\infty$.
2. $F(x)$ *is a strictly increasing differentiable function on* $[0,\bar{w}]$ *with* $0<\bar{w}<\infty$. *For* $x\leq 0$ $F(x)=0$, *for* $x\geq\bar{w}$ $F(x)=1$. *Further,* $0\leq w\leq\bar{w}$.

3a. $c(w)$ *is a continuous function on* $[0,\bar{w}]$.

3b. $c(w)$ *is a continuously differentiable function on* $[0,\overline{w}]$.
4a. $\forall\ 0\leq w<w^*\leq\overline{w},\ c(w^*) < c(w) + \frac{1}{\lambda}.(w^*-w)$.
4b. $\forall\ 0\leq w\leq\overline{w},\ c'(w) < \frac{1}{\lambda}$.

$c'(w)$ is the derivative of $c(w)$. Conditions 1, 2 and 3a are fairly general. Choosing zero to be the lower bound of the domain of $F(x)$ is a matter of convenience, we might as well choose another number. Likewise, $\overline{w}$ can be thought of as being a very large number. Conditions 4a and 4b assure that $c(w)$ does not increase too fast, in order to avoid the kind of problems related to the reservation wage property that were discussed before. Note that λ can be interpreted as an upper bound on the rate at which payment of transaction costs occurs. Consequently, the quantity $(w-\lambda c(w)).dt$ can be interpreted as the wage earned in a small time interval with length dt minus an upper bound on the expected amount of transaction costs to be paid in that small time interval, if the wage rate equals w. Conditions 4a and 4b state that this quantity must be increasing in w. Whether these are strong conditions cannot be said a priori but, as we shall see, it will turn up empirically. Note that Condition 4b is sensible only if Condition 3b also holds. Conditions 3b and 4b make it possible to give a characterization of the optimal strategy in terms of a differential equation. Condition 3b is not very strong since a differentiable function can approximate discontinuities well.

The proofs of the propositions are given in the appendix.

Proposition 1.

Let Conditions 1, 2 *and* 3a *be satisfied. Then* $R(w)$ *exists and it is the unique continuous function on* $[0,\overline{w}]$ *that solves equation* (2).

Proposition 2.

If in addition Condition 4a *is satisfied then* $R(w)$ *is strictly increasing on* $[0,\overline{w}]$.

This means that if c does not increase too fast as a function of w in the sense that Condition 4a is satisfied, then a high wage rate associated with a job implies a large (expected present) value of that job. From now on it is assumed that Conditions 1, 2, 3a and 4a hold. If $R(0)<R(w)+c(w)<R(\overline{w})$, then, because R is strictly increasing on $[0,\overline{w}]$, there exists a unique $\xi(w)$ such that

$$(3) \qquad R(\xi(w)) = R(w)+c(w)$$

while $R(x) \gtrless R(w)+c(w)$ if $x \gtrless \xi(w)$. Consequently, the optimal strategy for an individual earning a wage w then can be rewritten as follows: accept a wage offer x if $x>\xi(w)$ and reject it if $x<\xi(w)$. $\xi(w)$ is the reservation wage, which of course depends on all explanatory variables in the model. If $R(0) \geq R(w)+c(w)$ then any offer is acceptable for an individual earning w, so $\xi(w)$ may be anything ≤ 0; in that case we define $\xi(w)=0$. Similarly, for $R(w)+c(w) \geq R(\overline{w})$ we define $\xi(w)=\overline{w}$. Note that whether $\xi(w)=0$ or $\xi(w)=\overline{w}$ occurs for the range of w in the data set is an empirical matter. However, as we shall see, F(x) is not identified for our data and therefore neither is $\overline{w}$. We may assume that $\overline{w}$ is so large that for every relevant case (every individual in the data set) $R(\overline{w})>R(w)+c(w)$.

Because R is strictly increasing and continuous in its argument on $[0,\overline{w}]$, it follows that the inverse of R exists and in continuous in its argument on $[R(0),R(\overline{w})]$. Therefore $\xi(w)$ as defined above is continuous on $[0,\overline{w}]$. In summary, the optimal strategy satisfies the reservation wage property and the reservation wage ξ is a continuous function of w on $[0,\overline{w}]$. If Condition 4a is weakened by replacing the strict inequality by a weak inequality then we can only prove that R(w) is non-decreasing on $[0,\overline{w}]$. In that case there may be an interval $[\xi_1,\xi_2]$ with $0 \leq \xi_1 < \xi_2 \leq \overline{w}$ such that an individual is indifferent between acceptance and rejection of wage offers from that interval. Because there is a positive probability that wage offers $x \in [\xi_1,\xi_2]$ arrive, this arbitrariness would raise problems in any analysis of models in which such cases are allowed.

By strengthening Conditions 3a and 4a it is possible to derive expressions for the derivatives of R(w) and $\xi(w)$ with respect to w.

Proposition 3.

If Conditions 1, 2, 3b *and* 4a *are satisfied then* R(w) *is continuously differentiable on* $[0,\overline{w}]$ *and for every* $0 \leq w \leq \overline{w}$ *there holds that*

$$(4) \qquad R'(w) = \frac{1-c'(w)\lambda\overline{F}(\xi(w))}{\rho+\lambda\overline{F}(\xi(w))}$$

Proposition 4.

If Conditions 1, 2, 3b *and* 4b *are satisfied then* $R'(w)>0$ *on* $[0,\overline{w}]$ *and* $\xi(w)$ *is a continuously differentiable function of* w *for the wage intervals on* $[0,\overline{w}]$ *on which* $0<\xi(w)<\overline{w}$. *For those* w

$$(5) \qquad \xi'(w) = \frac{1+\rho\,.\,c'(w)}{1-\lambda\overline{F}(\xi(\xi(w)))\,.\,c'(\xi(w))} \cdot \frac{\rho+\lambda\overline{F}(\xi(\xi(w)))}{\rho+\lambda\overline{F}(\xi(w))}$$

Equation (5) can be rewritten by noting that the exit rate out of the present job equals

$$(6) \qquad \theta(w) = \lambda\overline{F}(\xi(w))$$

so

$$\xi'(w) = \frac{1+\rho\,.\,c'(w)}{1-\theta(\xi(w)).\,c'(\xi(w))} \cdot \frac{\rho+\theta(\xi(w))}{\rho+\theta(w)}$$

From the results so far the following corollary can be obtained.

Corollary.

Let Conditions 1, 2, 3b *and* 4b *be satisfied. Further, let* $\xi(w) \in <0, \overline{w}>$. *Then*

(i) $\quad \xi(w) \gtrless w \iff c(w) \gtrless 0.$

(ii) $\quad \xi'(w) \gtrless 0 \iff c'(w) \gtrless -\frac{1}{\rho} \iff R'(w) \lessgtr \frac{1}{\rho} \iff \theta'(w) \lessgtr 0.$

(iii) $\quad$ *if* $c'(w)=0$, *then*: $\xi'(w)<1 \iff c(w)>0.$

The results in (i) and (ii) make sense. If job changing costs are positive then one is more reluctant to move to another job than when such costs are absent. If c as a function of the wage level decreases very fast at w then the job offers that are not acceptable at w become acceptable for wages larger than w. Note that the model is not incompatible with an exit rate increasing with the present wage. The case $c'(w)=0$ for every w, $c>0$ has been analyzed extensively by Hey & McKenna (1979). In that case, if $\xi(w)<\overline{w}$ then $\xi(w)>w+\rho.c$ and the gap between $\xi(w)$ and w is a decreasing function of w. This can be understood by the following argument. Individuals take into account that they may change jobs more than once in the future. Therefore, the reservation wage has to exceed the sum of the present wage and the long-run compensation of the transaction costs that have to be paid for the first move. The more job changes one expects, the larger the gap between $\xi(w)$ and w because one does not want to pay too much transaction costs in order to reach a high wage level. Because the number of job changes one expects is relatively large for individuals who have a relatively small wage, this implies that the gap between $\xi(w)$ and w is decreasing in w.

In order to be able to use the model for structural empirical analysis the reservation wage has to be solvable for given w, c(w), λ, F(x) and ρ. It seems that in general the differential equation (5) cannot be solved analytically. Also, numerical methods may generate severe computational problems due to the lack of simple boundary values and the restricted interval ($0<\xi(w)<\bar{w}$) on which the equation holds. Therefore, a different route is taken in order to be able to calculate $\xi(w)$ as predicted by the model. Specifically, $\xi(w)$ is approximated by the first terms of a Taylor series expansion of $\xi(w)$ around c(w)=0, keeping w constant in the expansion.

Proposition 5.

Let Conditions 1, 2, 3b *and* 4b *be satisfied. Further, let* c(w) *be dependent on a parameter* η *such that* c(w)$-\eta$ *does not depend on* η *and* η *does not depend on* w (*η is an additive parameter of* c(w)). *Then for every* $w\in<0,\bar{w}>$

$$(7) \qquad \xi(w) = w + \frac{\rho+\theta(w)}{1-c'(w)\theta(w)} .c(w) + o(c(w))$$

The proof is in the appendix. The reservation wage is expanded as a function of the parameter η around $\eta = -(c(w)-\eta)$. For instance, if c(w) is a linear function of w, say $c(w)=c(0)+\alpha w$, then $\xi(w)$ is expanded around $c(0)=-\alpha w$. Note that ξ is expanded in terms of η only, with w being treated as an arbitrary constant. The resulting equation (7) suppresses the dependence of ξ on η and highlights the dependence of ξ on w. Several alternative expansions can be proposed, but these alternatives all have disadvantages. An expansion of $\xi(w)$ as a function of w around a specific value of w is impossible because we cannot calculate ξ for that value of w. Also, the expansion resulting in equation (7) is less stringent and has probably a better quality than an expansion of $\xi(w)$ as a function of all parameters of c(w) around the parameter values that correspond to c(w)=0 for every w. (In the example this would amount to expanding $\xi(w)$ around $c(0)=\alpha=0$ which is obviously more stringent than $c(0)=-\alpha w$). In the latter case the remainder generally is not $o(c(w))$.

From now on it is assumed that Conditions 1, 2, 3b and 4b are satisfied. Further, attention is restricted to cases in which $0<w<\bar{w}$ and $0<\xi(w)<\bar{w}$. The approximate $\xi(w)$ that is obtained by deleting the $o(c(w))$ term in equation (7) preserves many of the properties of the exact $\xi(w)$. (Note that $\xi(w)$ appears on both sides of equation (7) because θ depends on ξ. It can be shown that the implicit equation for the approximate $\xi(w)$ always has a solution.) For instance, if $c(w)>0$ then $\xi(w)>w$, if $c(w)=0$ then $\xi(w)=w$ and if $c(w)<0$ then $\xi(w)<w$. Further, if $c'(w)=-1/\rho$ then $\xi'(w)=0$, while if $c'(w)=0$ then there holds that $\xi'(w)\leq 1$ if and only if $c(w)\geq 0$. If, for every $w\in[0,\bar{w}]$, $c'(w)=-1/\rho$ then the

approximate and the exact reservation wage functions coincide. In summary, the approximation is good for values of c(w) which are not too large and it preserves many of the properties of the exact ξ(w). Though it would be interesting to know whether there is a systematic relationship between the explanatory variables in the model and the approximation error it is not possible to investigate this, since in general the exact ξ(w) cannot be solved. However, for very specific choices of F(x) and c(w) the exact ξ(w) can be solved and the relationship between w and the approximation error can be derived. Details are in the appendix. It appears that for values of F(x), c(w), λ and ρ that seem to be reasonable (as far as that is possible given that the exact ξ(w) has to be solvable), and for a wide range of w, the relative approximation error in ξ(w) is less than 0.4%.

The equation for the approximate ξ(w) has intuitive appeal. Suppose c'(w)=0. In that case ξ(w) is approximated by $w+(\rho+\theta(w)).c(w)$. As explained before, if c(w) is constant and positive then ξ(w) exceeds $w+\rho c(w)$ because one takes into account that one may have to pay transaction costs more than once in the future. Further, the more job changes one expects and the higher the transaction costs, the larger the gap between ξ and $w+\rho c(w)$. The term $\theta(w).c(w)$ in the approximation takes account of this. Now suppose c'(w) and c(w) are positive. Then in (7) ξ(w) exceeds the ξ(w) that would have prevailed if c'(w) were zero. This effect is more pronounced if θ(w) is large. Again this is plausible: if transaction costs increase with wages and if one is still at the bottom of the wage distribution then ξ(w) must be large to prevent that one has to pay too much transaction costs in order to reach a high wage level in the future. In this context it might be interesting to note that equation (7) can be rewritten as $\xi(w) = w + \rho.c(w) + \theta(w).c(\xi(w)) + o(c(w))$.

4.2.3. *Job durations*

As shown before, the exit rate out of the present job θ(w) equals $\lambda\overline{F}(\xi(w))$ and therefore depends on all structural parameters λ, F(x), c(w), w and ρ. However, because of the stationary assumption θ(w) does not depend on the elapsed duration in the present job. Consequently, the job duration has an exponential distribution with parameter θ(w). In Section 4.5 the validity of the stationarity assumption will be tested.

4.3. The data

4.3.1. *The data set*

The data set used is constructed from the Labour Market Research Panel, a survey conducted by the Netherlands Organization for Strategic Labour Market Research (OSA). As of April 1985 a sample of about 4000 individuals living in The Netherlands is interviewed every one and a half year. The sample includes only individuals aged between 15 and 61 and it over-represents individuals who are in certain labour market states at the date of the first interview (notably employment and unemployment) but it is supposed to be random in all other respects. For our study only the first wave of the panel is available. Respondents are asked to recall their labour market history from January, 1980 until the date of the interview. Further, they were asked to provide information on their income at the date of the interview. The data set contains a wide range of job characteristics and information on the social and working environment of individuals who are employed in April, 1985. This makes the data set particularly useful for explaining individual differences in job durations. Another distinguishing feature of the data set is that individuals who were employed at the date of the interview were asked for their lowest acceptable net wage offer. Responses on this question are interpreted as the observed counterpart of the reservation $\xi(w)$.

For our estimation purposes we selected individuals who were employed in a paid job at the date of the interview. Since we do not know the income, the working environment and the job characteristics associated with previous jobs, we cannot use job spells that ended before April 1985 for the empirical analysis. Self-employed individuals are deleted because their labour market behaviour may deviate substantially from the behaviour of employees. For reasons that will be explained in the next subsection attention is restricted to individuals who are aged over 22 at the date of the interview. As a result we obtained a sub-sample containing 1757 individuals. The elapsed job duration is constructed by determining for how long the individuals were employed in the present job. Of these job durations, 66% are censored in the sense that it is only known that the realized elapsed duration exceeds 5.4 years. From all 1757 individuals, 1461 (83%) gave an answer to the question about their reservation wage. Using a standard test we did not find a significant difference in the mean wage of respondents and nonrespondents on that question. This result is interpreted as favouring the assumption that per-period search costs do not matter and that nobody precludes transitions to another job. Figure 1 gives a scatter diagram of all 1461 wage, reservation wage points (measured in Dutch guilders per month). For only 5% of the individuals the observed reservation wage is smaller than the present wage. It is clear that there is a positive relationship between the variables.

4.3.2. *The likelihood*

Information on elapsed job durations and observed reservation wages can be used to estimate the model. By assuming that the individual entry rate into the present job is constant before the moment of the interview and that $\xi(w)<\overline{w}$, i.e. $\theta(w)>0$, the elapsed job duration t has an exponential distribution with parameter $\theta(w)$ (see e.g. Ridder (1984)). The entry rate may depend on the wage of the present job and other observables without affecting this result. Young individuals, of course, have had only a few years to enter a job, so the constant entry rate assumption does not hold even approximately for them. Alternatively, if one views the labour market as a Markov process then one might say that for young individuals the process is not (approximately) in equilibrium because the origin of the process is only a couple of years before the point of observation. Because of this it was decided to delete all individuals aged below 23 from the sample. Let $d_1=1$ if t is censored and $d_1=0$ otherwise. The part of the individual log-likelihood contribution $\mathcal{L}$ due to the elapsed duration t is $\mathcal{L}_1$,

$$(8) \qquad \mathcal{L}_1 = (1-d_1).(\log \theta(w)) - t.\theta(w)$$

The observed reservation wages are denoted by $\tilde{\xi}(w)$. These may differ from the true reservation wages

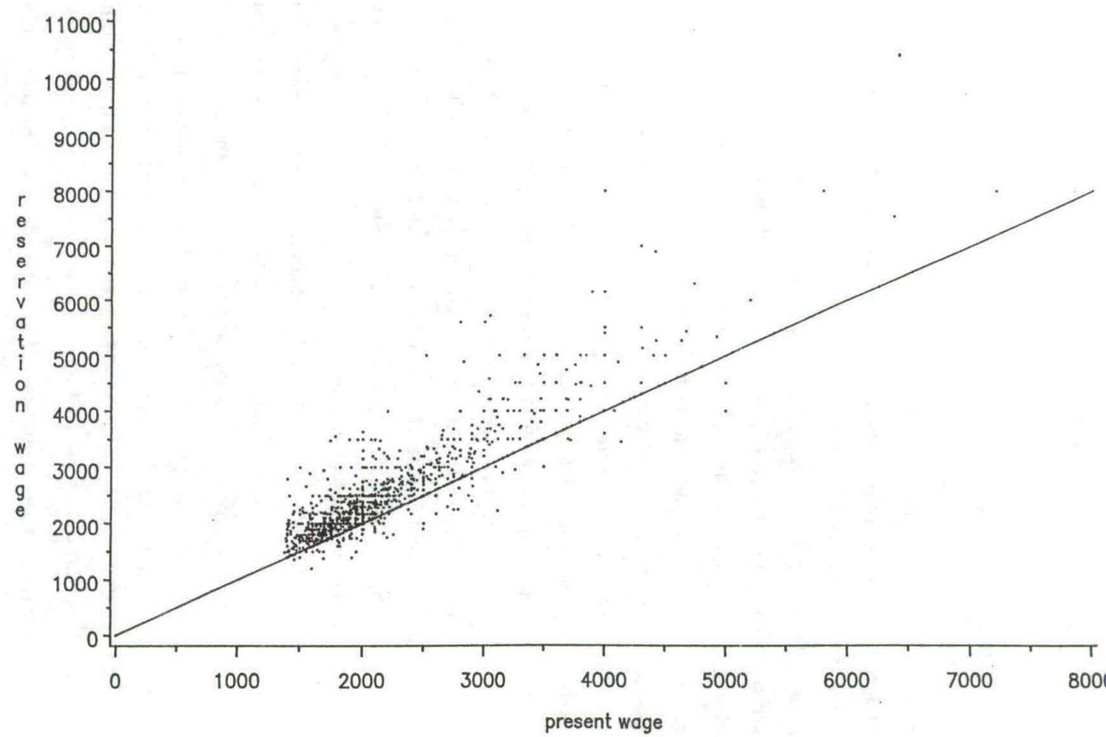
$$(9) \qquad \tilde{\xi}(w) = \xi(w) + \varepsilon$$

ε is an error term which is interpreted as a measurement error that is i.i.d. across individuals and independent of duration t and present wage w. Consequently, individuals use $\xi(w)$ instead of $\tilde{\xi}(w)$ as their strategy so θ depends on ξ instead of $\tilde{\xi}$ and equation (8) does not depend on ε. In addition, t and $\tilde{\xi}$ are independent. By assuming that the distribution of ε belongs to some parametric class (e.g. normal) the part of $\mathcal{L}$ due to $\tilde{\xi}$ can be constructed. Let $g(\varepsilon)$ be the p.d.f. of ε.

$$(10) \qquad \mathcal{L}_2 = (1-d_2) .\log\ g(\xi(w)-\tilde{\xi}(w))$$

FIGURE 1

reservation wages of working individuals



with d_2=1 if $\tilde{\xi}$ is missing and 0 otherwise. The individual log-likelihood contribution is given by the sum of $\mathcal{L}_1$ and $\mathcal{L}_2$. The structural parameters and functions of the on-the-job search model (λ, F(x), c(w) and ρ) enter the likelihood via $\theta(w)$ in $\mathcal{L}_1$ and $\xi(w)$ in $\mathcal{L}_2$. The true reservation wage $\xi(w)$ is the solution of equation (7), deleting the $o(c(w))$ term in that equation.

$$(11) \qquad \xi(w) = w + \frac{\rho+\theta(w)}{1-c'(w)\theta(w)} .c(w)$$

When taking a closer look at the likelihood equations one sees that not all parameters are identified using data on t and $\tilde{\xi}$. In particular λ and F are not identified because both (8) and (11) only depend on the product $\lambda\overline{F}(.)$. This is also a commonly encountered problem in unemployment duration analysis using structural models (see e.g. Flinn & Heckman (1982)), and it basically results from the fact that only acceptable wage offers matter for the optimal strategy and the exit rate out of the present state. The general approach to obtain identification of a structural job search model is to assume recoverability of F and use data on post-spell wages. Because our data set is essentially a cross-section it does not provide information on wages that are earned after moving to another job than the present one so this approach cannot be used here. Alternatively, the present wage itself could be regarded as a drawing from the wage offer distribution, truncated at the value of the reservation wage at the previous job or at the spell of unemployment that preceded. However, we do not know anything about the values of these points of truncation, simply because the data set does not provide income variables for spells that ended before the date of the interview. Consequently, it seems that the model cannot be estimated.

However, as was set out in the previous sections, our main empirical interest is in factors that obstruct flexibility of the labour market and in particular in the costs associated with moving to another job. That is, c(w) is the 'parameter' of interest. Now note from equations (10) and (11) that c(w) is identified from the data on $\tilde{\xi}$ if θ is known. But θ is identified from the data on t (see equation (8)). Of course, θ depends on λ, F and ξ. However, we can do a reduced-form estimation of θ from (8) and use these estimates in the reservation wage equation (11) in order to estimate c(w). Such an estimation method does not require identification of λ and F but yet uses the theoretical framework of on-the-job search theory to interpret the reservation wage data. One may say that the method is flexible in the sense that identification of a structural parameter of interest is achieved without the need to make strong assumptions on certain other structural parameters and functional forms. Moreover, by using a reduced-form specification for $\theta(w)$ we are able to check whether certain predictions of the theory hold. For instance the theory predicts $\theta'(w)<0 \Leftrightarrow \xi'(w)>0$. A fully structural empirical specification of $\theta(w)$ imposes such restrictions and therefore makes such empirical checks impossible. In Subsection 4.3.3. which deals with parameterizations of $c(w)$ and $\theta(w)$ some other predictions from the theory are derived.

In Section 4.2 it was mentioned that the on-the-job search model could be given an alternative interpretation in terms of vacancy offers instead of job offers. However, in that case only λ and F obtain another meaning and because λ and F do not appear separately in the empirical specification this implies that there are no interpretation differences for the empirical results.

One can argue that because we use the approximation from Proposition 5 for the true reservation wage or because the model is not well specified even if there is no approximation error in $\xi(w)$, we actually make a specification error. Specifically, $\xi(w)$ from equation (11) may not be the true reservation wage. The exit rate θ then depends on the unknown true reservation wage but since θ is estimated in a reduced-form this is no problem. Further, ε in equation (9) then represents the sum of a specification error and a measurement error. Assume that the specification error is independently distributed across individuals and is independent of duration t. We can then still use the estimation method proposed above. (Note that an empirical analysis of a fully structural model would (apart from the identification problems) become very complicated if one allows for errors in the specification of $\xi(w)$.)

4.3.3. *The empirical implementation*

Now that we have specified the empirical setting and described the data we discuss in this subsection parameterizations and the choice of explanatory variables. The cost of moving to another job $c(w)$ is written as a linear function of explanatory variables x_1 and the present wage on the relevant wage interval,

$$(12) \qquad c(w) = x_1'\gamma_1 + \alpha.w$$

Such a specification satisfies the assumptions of Proposition 5 for any finite $\alpha<1/\lambda$. The vector x_1 includes characteristics of the neighbourhood in which one lives, personal characteristics and characteristics of the present job.

The latter can be subdivided into occupation dummies and pecuniary and non-pecuniary fringes.

The exit rate out of the present job $\theta(w)$ is written as an exponential function of explanatory variables x_2 and the logarithm of the present wage

$$(13) \qquad \theta(w) = \exp(x_2'\gamma_2 - \beta \,.\log w)$$

This specification is in accordance with specifications in the literature on reduced-form hazards out of unemployment in that log θ depends on log income and is a linear function of x. We assume that (13) gives a good approximation of (6) for the range of x and w in the data. From equation (6) it follows that θ depends on λ, F and ξ and therefore also depends on c(w). Consequently, x_2 has to include all explanatory variables in x_1. Most of these explanatory variables also influence λ and F. For instance the age of an individual or whether he is married may influence c(w) but may also give an indication of the productivity of the job searcher and therefore influence λ.

According to the theory the parameters γ_1, γ_2, α and β are interrelated and the estimation results can be used to check such interrelations. First, $\alpha<1/\lambda$ has to hold in order to be sure that one can safely use equation (11). This cannot be checked because λ is unidentified, but since $\theta\leq\lambda$ a necessary condition for $\alpha<1/\lambda$ that can be checked is that for every individual the estimate of α has to satisfy $\alpha<1/\theta$. From equation (6) it follows that θ depends on w by means of ξ, so the sign of $\theta'(w)$ must be opposite to the sign of $\xi'(w)$. From (13), β and $\xi'(w)$ must have the same sign. The empirical model specification consists of the equations (11), (12), and (13). By differentiating ξ it can be shown that the relation between $\xi'(w)$ and β does not necessarily follow from the empirical specification and can be checked after estimation. Another interrelation predicted by the theory is about γ_1 and γ_2. Consider an explanatory variable that influences c(w) while it can be safely assumed that it does not influence λ, F or ρ. In the appendix it is shown that according to the theoretical model (equations (6), (11) and (12)) the signs of the parameters in $\theta(w)$ and c(w) associated with that variable must be opposite if $\alpha>-1/\rho$ and $\xi'(w)>0$, or $\alpha<-1/\rho$ and $\xi'(w)<0$. If the variable has a positive effect on the cost of moving to another job then it must have a negative effect on the exit rate to another job. Again this can be checked by comparing the estimates of γ_1 and γ_2.

The estimation procedure is as follows. First, β and γ_2 are estimated by ML using the BHHH algorithm. The likelihood function is (8) with (13) substituted for $\theta(w)$. (Note that we assume that there is no unobserved heterogeneity of $\theta(w)$ in the sample. In Section 4.5 this assumption is tested.) Next α and γ_1 are estimated by nonlinear least squares. The objective function follows from (9) and equals

$$\sum_{i=1}^{n} (\tilde{\xi}_i - \xi_i)^2 \qquad n=1461$$

ξ_i is calculated from equation (11). We plug in the individual predicted $\theta(w)$from the first estimation step and substitute (12) into (11). Note that by using this estimation procedure we do not have to make strong assumptions on the distribution of the error term ε. It is assumed that the distribution of ε does not depend on the explanatory variables w and x. However, in the previous subsection it was argued that the difference between the exact $\xi(w)$ in the model and the approximation of it (equation (11)) may be part of ε and in the appendix it is shown that for reasonable parameter values the approximation error is decreasing in w. Still, the approximation error is always very small and is therefore probably only a minor part of ε, so we do not expect it to cause a severe violation of the independence assumption on ε and w. The variance σ^2 of ε is the variance of the sum of the specification error and the measurement error. The subjective rate of discount ρ is fixed at 15% a year. In Section 4.5 we examine the robustness of the results with respect to the numerical value of ρ.

4.4. Results

4.4.1. *Parameter estimates*

The parameter estimates for the model described in Section 4.3 are presented in the Tables 1 and 2. The unit time period is one month. For the age and occupation dummies the reference categories are the age category 41–60 and the occupation category of scientists, engineers and artists. Though β and γ_2 are estimated prior to α and η_1 we start by discussing the results for the latter because those are structural parameters.

Housing accommodation circumstances have a strong influence on job changing costs. If one expects it to be hard to sell the present house or to find another house to rent when moving to another job, then job changing costs are (significantly) larger than when such problems are not expected. Further, if the distance between house and working place is less than 10 km, then job changing costs are larger, indicating that one is reluctant to give up the advantage of short travelling times between home and work.

Table 1. Parameter estimates for the costs associated with changing jobs.

variable	coefficient	(t–ratio)
constant	3233	(1.0)
small distance home/work	2410	(2.4)
civil servant	199	(0.2)
fringe benefits	851	(3.0)
attached to environment (social)	–109	(0.1)
married	–586	(0.5)
housing problems expected	2435	(2.6)
unsatisfied with job (non–pecuniary)	–4290	(2.3)
occupation administrative/commercial	3302	(2.6)
occupation services	3614	(2.3)
occupation farm–labourer/industrial	–416	(0.3)
aged below 30	–13865	(9.0)
aged between 30 and 40	–8915	(5.7)
log (# working in household)	61	(0.0)
wage	6.37	(7.7)
standard error σ	396	
1461 observations		

Both pecuniary and non–pecuniary job characteristics are strong determinants of c. If the numbers of fringe benefits categories is large then transaction costs are high, whereas if one is very dissatisfied with the present job from a non–pecuniary point of view the opposite holds. The correlation coefficient of the present wage and the non–pecuniary satisfaction variable equals –0.02 so there is no multicollinearity effect here. One might argue that job characteristics do not show up in labour market behaviour just as elements of job changing costs but are in fact properties of a job that have intrinsic utility. However, that would imply that a job is represented by a vector of characteristics rather than by just a wage and the optimal strategy would be multidimensional and generally unsolvable. Also, certain job characteristics like special fringe benefits can be obtained only after having worked in a job for some time so a job transition implies a temporary loss of them which can therefore be represented as a transaction cost.

The occupation dummies show large differences. Apparently individuals who have an administrative job or who work in the commercial or services sector face high job changing costs. This may be because the possible loss of pension claims is high in these sectors or that in these sectors institutional restrictions discourage individuals to move to a competing company. It seems that the kind of occupation one has has more influence on c than whether one is a civil servant or not. Age is a very important and significant determinant of c. Both the accumulation of job-specific human capital and psychological factors like an increased attachment to the neighbourhood in which one lives may be responsible for the high c for older individuals. Also, the amount of pension claims increases with age. Finally, there may be a finite horizon effect which is not included in the model. Assume that there is a point of time T (say the retirement age) after which one cannot work and assume that $c(w)>0$ for every w. Then for points of time sufficiently close to T it is optimal to reject every possible offer. This is basically because the period of time that can be used to earn back $c(w)$ decreases as time proceeds. The age coefficient in $c(w)$ for older individuals may be biased upward because of this. Note that the argument implies that in that case the corresponding coefficient in θ is biased downward.

Rather surprisingly, other personal characteristics like marital status and number of working individuals in the household do not have influence on c. Also, it seems that it does not matter whether one is attached to the social life in the present environment. One (rather devastating) explanation of this result is that the 'reservation wage' question was (mis)understood to refer to jobs in the present neighbourhood only. Another possible explanation is that individuals who are more attached restrict the job search to their present neighbourhood. In both cases one would expect that the variable under consideration does have a significant effect on θ.

The coefficient α related to the present wage w is very significant. Probably the wage variable takes account of many factors some of which are mentioned in Section 4.2, because w influences the magnitude of the effects of those factors on c. The estimated standard deviation of ε is quite large as compared to the average value of ξ, which is about 2530. This confirms the supposition that the error due to using the approximation of the exact ξ is only a small part of ε.

Table 2. Parameter estimates for the transition rate from one job to other jobs.

variable	coefficient	(t–ratio)
constant	–1.59	(1.1)
small distance home/work	–0.34	(3.8)
civil servant	–0.36	(3.9)
fringe benefits	–0.12	(4.6)
attached to environment (social)	–0.17	(2.0)
married	–0.18	(1.7)
housing problems expected	–0.09	(1.1)
unsatisfied with job (non–pecuniary)	0.51	(2.4)
high education	0.28	(2.6)
occupation administrative/commercial	–0.50	(4.3)
occupation services	–0.10	(0.7)
occupation farm–labourer/industrial	–0.40	(3.3)
aged below 30	1.69	(13.0)
aged between 30 and 40	0.86	(6.7)
log (# working in household)	0.24	(2.0)
log (wage)	–0.41	(2.2)

Log–likelihood = –3426.84

1757 observations

Table 2 presents the parameter estimates for the exit rate out of the job θ. Note that explanatory variables entering θ represent determinants of λ, F and c. The present wage w has a negative influence on the exit rate. This confirms the prediction of the basic theory of on–the–job search, though our extended on–the–job search model would not be incompatible with a non–negative coefficient either. The common on–the–job search interpretation of the positive correlation of wages and tenure is that once a job with a high wage is obtained it is hard to find an even better job, so a high wage causes tenure. In our model a high wage also implies high job changing costs which makes individuals earning a high wage even more reluctant to change jobs.

Other factors that probably influence θ by way of c are factors related to housing and the non-wage characteristics of the present job. These variables all have the right sign in θ and are all significant except for the variable indicating whether housing problems are expected when one needs to move (which has the right sign but is insignificant). The occupational dummies probably reflect differences in λ and F across different segments of the labour market. Civil servants have a lower exit rate, which may indicate, for instance, that the arrival rate or the variation in wage offers is smaller for them. Higher educated individuals have a higher exit rate, which may indicate a higher arrival rate or higher variation in wage offers. Whether one is attached to the social life in the present social environment has a significant influence on job duration but not on job changing costs. As suggested in the previous paragraph, it may be that individuals who are more attached restrict the search for better jobs to their present geographical area only and do not experience high c by changing jobs within that area. Note that this argument cannot be deduced from our model. The number of working individuals in a household has a positive influence on θ (but no influence on c). The same effect for the exit rate out of unemployment was observed in Chapter 2. It may be a consequence of a positive relation between unobserved characteristics of the individual under consideration and characteristics of other household members, as far as these characteristics are relevant for employers. It may also reflect the fact that if the number of working household members is large then one has easier access to employers.

4.4.2. *The characteristics of the search process*

Given the parameter estimates the main variables of the search process can be estimated and the influence of changes of the explanatory variables on these main variables can be evaluated. Table 3 presents sample averages of w and of the estimates of ξ, θ, c and $d\xi/dw$ for different age categories. Recall that ξ denotes the (approximated) true reservation wage as given in equation (11). The sample average of ξ is about 14% higher than the sample average of w for every category. The variance of ξ across the sample is fairly large, especially for older individuals. The minimum estimated ξ equals 1370 which is slightly below the legal minimum wage (1450). The derivative of ξ with respect to w is positive for all individuals and is on average slightly larger than one. The latter is a consequence of $\alpha>0$. Figure 2 gives ξ as a function of w for an individual with average values for the explanatory variables. The function is almost linear. However, by choosing different values of the explanatory variables the location of the function may change.

FIGURE 2

estimated reservation wage as a function of present wage

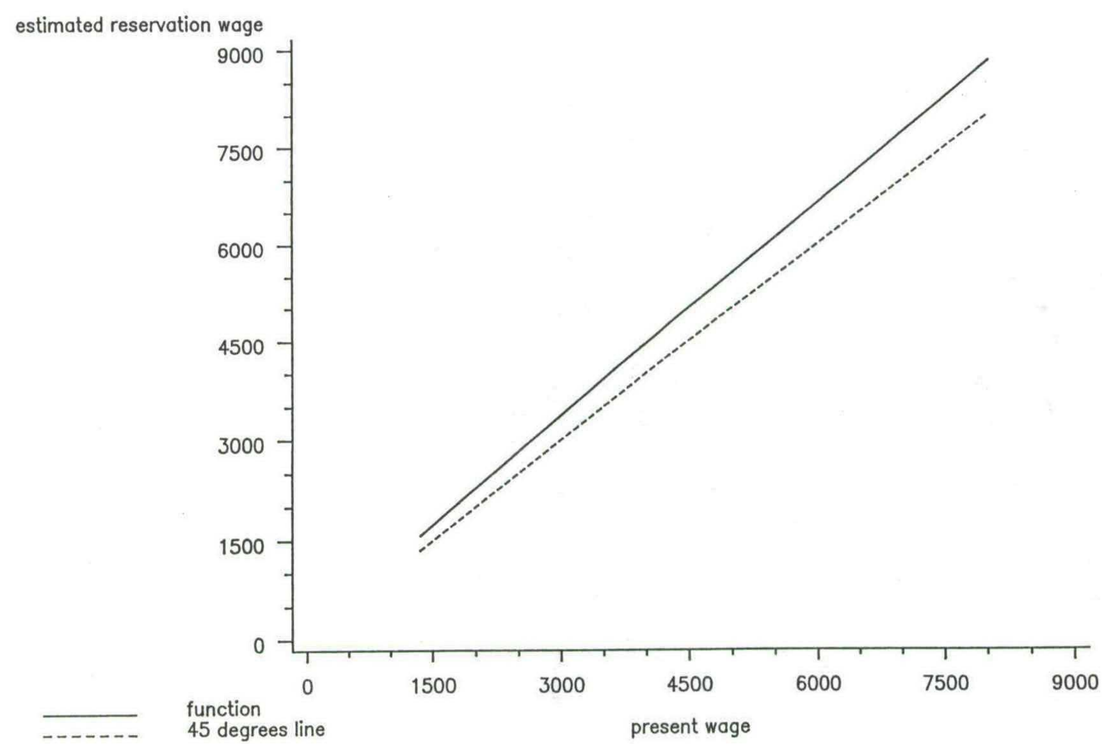


Table 3. Characteristics of the job turnover process.

age category	23–29	30–40	41–65	average
w (wage)	1882	2304	2430	2231
	(412)	(721)	(843)	(733)
ξ (reservation wage)	2121	2595	2798	2534
	(487)	(802)	(914)	(817)
θ (exit rate out of the present job)	0.0161	0.0062	0.0025	0.0077
	(0.0071)	(0.0029)	(0.0010)	(0.0068)
c (costs of changing jobs)	8136	15866	25508	17015
	(3670)	(5248)	(5804)	(8502)
$d\xi/dw$	1.17	1.10	1.08	1.11
	(0.06)	(0.02)	(0.00)	(0.05)

the unit time period is one month.
in parentheses: the standard deviation of the variable across all 1757 observations.

Generally, older individuals have both a higher wage and higher transaction costs (which is partly due to the higher wage). Note that on average an individual aged 50 faces job changing costs that are about three times as large as the job changing costs for someone aged 25. As a result, older individuals are more selective in their search for a better job and have much smaller exit rates.

The results so far enable us to investigate a number of questions related to the effectiveness of policies aimed at an increase of job mobility. In particular we are able to examine the effects of changes in the level of job changing costs and changes in the job offer arrival rate by calculating several elasticities. Table 4 presents sample averages of the elasticities of the reservation wage and the transition rate to other jobs with respect to the present wage, the job offer arrival rate and the level of job changing costs. Because the values of the elasticities do not differ substantially across different age categories we only present averages over the whole sample. The elasticities with respect to w can be calculated directly from the estimated model. It is impossible to calculate the elasticities with respect to λ and c(w) using the empirical model specification only because the equation for θ (equation (13)) does not show how it depends on c(w) and λ. Moreover, λ itself is unidentified. However, by assuming that (13) represents the theoretical equation (6) we can derive the elasticities, as is shown in the appendix. Unfortunately the method cannot be used to derive elasticities with respect to parameters of F. The elasticities with respect to c(w) are derived conditional on the present wage and on α, so it is assumed that w and α do not change if c(w) changes. For individuals who had c(w)<0 the elasticities with respect to c(w) were calculated by defining $\partial \log c(w) = \partial c(w)/|c(w)|$.

Table 4. Elasticities.

(i) *of the reservation wage* ξ		
$\frac{\partial \log \xi}{\partial \log w}$	(with respect to the wage)	0.98
		(0.06)
$\frac{\partial \log \xi}{\partial \log \lambda}$	(with respect to the job offer arrival rate)	0.04
		(0.03)
$\frac{\partial \log \xi}{\partial \log c(w)}$	(with respect to the level of job changing costs)	0.12
		(0.03)
(ii) *of the transition rate from one job to other jobs* θ		
$\frac{\partial \log \theta}{\partial \log w}$		-0.41
		(0.00)
$\frac{\partial \log \theta}{\partial \log \lambda}$		0.98
		(0.01)
$\frac{\partial \log \theta}{\partial \log c(w)}$		-0.05
		(0.01)

in parentheses: the standard deviation of the elasticity across all 1757 observations.

The elasticity of θ with respect to c(w) is very small. This can be explained as follows. First, because for a wide range of w the elasticities of θ and ξ with respect to w are about -0.4 and 1 respectively, it follows that for a wide range of ξ the elasticity of θ with respect to ξ is also about -0.4. This means that F has a very long tail and small changes in ξ do not affect θ very much. Secondly, note that if c(w) were zero then ξ(w) would equal w, so changing c(w) can only affect the gap between ξ(w) and w. In our sample this gap is on average about 12% of ξ(w). Consequently, an increase of c(w) will not make the individual much more selective with regard to job offers. Because c(w) influences θ by way of ξ these two arguments imply that θ is insensitive with respect to small changes in c(w). The effect of λ on θ is both direct and indirect (via ξ). The direct effect by definition has elasticity one and clearly dominates the indirect effect, which can only influence ξ by affecting the gap between ξ(w) and w.

In the absence of comparable studies it is hard to say whether these results are common or not. However, it should be noted that the data used to estimate the model are from a period in which there was a very slack labour market (see Chapter 2), so it may well be that the elasticities have different values in other circumstances. In particular, if λ is larger, then more job-to-job transitions are possible in the future, which implies that ξ is larger and that ξ and θ are more sensitive with respect to changes in c(w). Also, one can show that if ρ or c(w) are larger then ξ and θ are more sensitive with respect to changes in c(w). However, though the sample contains individuals with estimated c(w) as large as 50000, the estimated absolute value of the elasticity of θ with respect to c(w) never exceeds to 0.10.

From the results of Table 4 one may conclude that if one is interested in an increase of job mobility then an increase of λ is more effective than a decrease of c. It is hard to say which explanatory variables influence duration mainly through λ and which exert their influence through c. From the reduced-form estimation results for θ one can examine the effect of changing an explanatory variable on the expected duration. For instance, from Table 2 it follows that if the number of fringe benefits categories decreases by one then the expected duration of the job (which is one over θ) decreases by 11%. Also, if someone changes from being a civil servant to not being a civil servant then the expected duration of the job decreases by 30%.

4.5. The model specification revisited

In this section it is examined whether the estimation results satisfy non-imposed properties of the theoretical model and whether the results are sensitive with respect to changes in some of the assumptions made.

In Subsection 4.3.3. it was shown that $\alpha<1/\theta$ is necessary in order to have a sensible empirical model specification. As $\alpha=6.37$ while the smallest estimate of $1/\theta$ in the data set equals 21 it follows that this condition is satisfied for all individuals in the data set. Also, the theory predicts that β and $\xi'(w)$ have the same sign. From the results, β is positive (0.41) and the smallest estimate of $\xi'(w)$ in the data set equals 1.07 so the prediction is validated. Another prediction from the theory is that if $\alpha>-1/\rho$ and $\xi'(w)>0$ (which both hold according to the results) then the signs of parameters associated with explanatory variables x that only influence c(0), are opposite for θ and c. It is hard to say which x influence c only but of all 13 explanatory variables in c(0) 9 satisfy this restriction while the others (typically variables that represent household circumstances) are not significant for c. Thus, the reservation wage assumption is not invalidated and there are no inconsistencies in the results.

As said before, the rate of discount ρ is fixed at 15%. This is done basically because the data are not able to distinguish between ρ and the constant term in c(0). In order to examine the robustness of the results with respect to the value of ρ the model is re-estimated with different ρ, specifically with ρ=10% and ρ=25% a year. Note that the estimation results of θ do not depend on ρ at all. The estimates for c do change although sign and significance generally remain preserved. The resulting sample average of c equals 12012 if ρ=25% and 21989 for ρ=10% (it is 17015 if ρ=15%). The value of c for older individuals in particular is very sensitive to alternative values of ρ. However, for all categories the values of the resulting characteristics ξ and $d\xi/dw$ and the various elasticities are almost completely insensitive to ρ. Also, the estimate of σ does not change when varying ρ. Consequently, the main results are unaffected by the value of ρ.

One may argue that gender should be included as a separate regressor in c and θ besides the other regressors indicating personal characteristics. However, not surprisingly, a 'gender' dummy variable is highly correlated with the 'married'-dummy variable and with the variable related to the number of working individuals in the household. Inclusion of a 'gender' dummy variable in c and θ does not have notable consequences for the main results. The variable itself is insignificant for θ but the transaction costs are significantly higher (2669 guilders, t=2.5) for men than for women. The latter may arise because the other variables that indicate personal characteristics may be misspecified, or because females may restrict attention to jobs in the present neighbourhood more than men do. Note that θ for females is over-estimated if for this category the transition rate into nonparticipation is not negligible.

From the discussion of the elasticity estimates it is clear that the elasticity of θ with respect to ξ is an important determinant of those estimates. The elasticity of θ with respect to ξ is determined from the specifications of θ and ξ as functions of w. These in turn are of course completely determined by equations (11), (12) and (13). In order to examine whether the elasticity estimates are sensitive with respect to the (restrictive) specifications of θ and c as functions of w the model is re-estimated using more flexible functional forms for $\theta(w)$ and $c(w)$. Specifically, the variables $(\log w)^2$ and w^2 are included as additional regressors in x_2 and x_1, respectively (see equations (13) and (12)). Adding w^2 in $c(w)$ has almost no effect on the estimation results, whether $(\log w)^2$ is included in $\theta(w)$ or not. In both cases $c(w)$ is virtually linear on the wage interval of interest. If $(\log w)^2$ is included in $\theta(w)$ then the estimates of the coefficients in $\theta(w)$ associated with log w and $(\log w)^2$ are –17.79 (t=3.0) and –1.112 (t=3.0), respectively. This means that $\theta(w)$ attains a minimum at w=2970, so for individuals who have w>2970 (12% of the sample) $\theta(w)$ is increasing in w. However, if attention is restricted to the subsample for which w>2970, then it appears that $\theta(w)$ is not increasing on w>2970. The fact that the $(\log w)^2$ term in $\theta(w)$ is significantly negative may therefore be due to the non-linearity of $\log \theta(w)$ in log w for small w. The estimates and standard errors of the other parameters in $\theta(w)$ and $c(w)$ and the main characteristics of the search process are almost identical to those in Tables 1, 2 and 3. Because of the inclusion of the $(\log w)^2$ term in $\theta(w)$, the sample average of the elasticity of θ with respect to w changes from –0.41 to –0.73. Since the elasticity of ξ with respect to w does not change much, this implies that on average θ is now more sensitive with respect to ξ. As a result, the average elasticities of θ with respect to c and λ are now –0.10 and 0.96, respectively. However, for individuals who have w>2970, $\theta'(w)>0$ and therefore the estimates of the elasticities of θ with respect to ξ and c are positive, which is of course in conflict with the theory. If, following the argument above, this result is regarded as a consequence of the rigidity of the quadratic specification of $\log \theta(w)$ as a function of log w, then this means that the sample averages of the elasticities of θ with respect to c and λ are over-estimated (that is, they are <–0.10 and <0.96, respectively). The standard errors of the elasticities in the sample equal 0.10 and 0.04 respectively. For some individuals in the sample (most of which have low wages) the elasticity of θ with respect to c is as small as -0.40. The estimates of the elasticities of ξ with respect to c and λ are identical to those in Table 4. In sum, the main conclusions from Section 4.4 seem to be robust with respect to the specifications of θ and c as functions of w, although the ineffectiveness of a reduction of c as a tool in stimulating job mobility is less extreme if a more flexible specification is used.

When estimating the model, no account has been taken of unobserved heterogeneity in the sample. This may in particular be a problem for the estimation of θ, because in duration models the estimates are inconsistent if unobserved heterogeneity is present in reality even if the heterogeneity is orthogonal to the included explanatory variables. Therefore we tested the assumption of no heterogeneity in θ. Specifically, we re-estimated θ, assuming that there is an unobserved heterogeneity term v that acts multiplicatively on θ and is independent of t. First assume that v in the stock of employed at the moment of the interview has a normal distribution that is independent of x (this is sensible only if $P(v<0)$ is very small) with variance σ^2. Testing the assumption of no heterogeneity means testing $\sigma^2=0$. We find $\hat{\sigma}^2=0.16$ with t-value 1.0 so a Wald test does not reject $\sigma^2=0$ ($P(v<0)$ is smaller than 0.01). Alternatively, let v in the stock have a gamma distribution independent of x (this is equivalent to assuming that (i) v in the inflow has a gamma distribution, (ii) the inflow rate into the present job factorizes in terms of v and x and (iii) v and x in the population are independent) with variance σ^2. Then $\hat{\sigma}^2=0.14$ (t=1.4) and the estimated variance of v in the inflow equals 0.16 (t=0.9) so again the Wald test does not reject. Though these results may seem a little surprising, recall that the set of observed explanatory variables is rather unique in that it contains an indicator of non-material job satisfaction and other characteristics of the (working) environment. As Holmlund & Lang (1985) point out these variables are generally unobserved which may cause (spurious) negative duration dependence of θ.

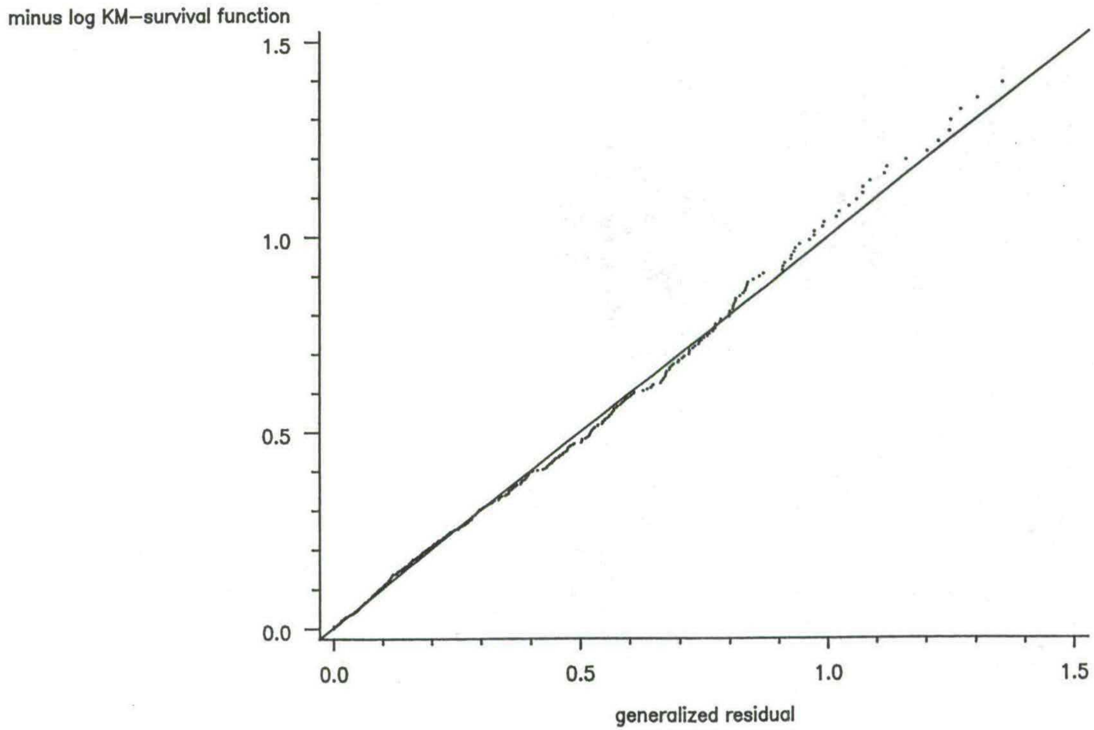
According to various theories of labour market behaviour of employed individuals, notably human capital theory and job matching theory, some of the basic assumptions of on-the-job search theory do not hold. It is argued that wages are not (approximately) constant within jobs and that the exit rate out of a job is truly duration dependent (see e.g. Mortensen (1986), Lancaster, Imbens & Dolton (1987), Mortensen (1988)). Though the former conjecture cannot be investigated given the data used, the latter one can. We examine the assumptions that θ is constant in a non-parametric way, in order to avoid the risk of not detecting certain (non-monotonic) alternatives. If θ is constant, then $z=t.\theta$ has an exponential distribution with parameter 1, so minus the log empirical survival function of the so-called generalized residuals $t.\hat{\theta}$ should approximately be a 45° line, $\hat{\theta}$ being the estimate of θ that was obtained before. If θ is not constant as a function of job duration then this result does not hold. Ridder (1987) examines for specific departures how the plot for the minus log empirical survival function of $t.\hat{\theta}$ is affected. In fact this method can also be used to detect unobserved heterogeneity. Though Ridder's (1987) results apply to complete durations they can also be used for elapsed durations from a stock sample because of the intimate links between the distributions of these durations (see e.g. Ridder (1984), these links hold conditional on the constant-entry rate assumption). Figure 3 gives the minus log of the Kaplan-Meier estimator of the survival function of the generalized residuals. Note that every point corresponds to an uncensored observation of the job duration. For 78% of all 600 points the generalized residual is smaller than 0.5. The plot shows that the minus log survival function closely follows the 45° line. Apparently job durations can be described fairly accurately by an exponential distribution that depends on the explanatory variables we used. Note however that departures from exponentiality may be obscured because we use estimates of θ from a fitted parameterized model. For instance, if there is duration dependence due to job-specific human capital accumulation, then w is an increasing function of duration and the estimate of the coefficient in θ associated with w will pick out some of the duration dependence.

Another property of job matching models and models of job-specific human capital accumulation is that in those models the reservation wage is smaller than the present wage (see Jovanovic (1979), Jovanovic (1984), Mortensen (1988)). In our sample however the reservation wage is on average 14% larger than the present wage, and for only 5% of the sample the (observed) reservation wage is smaller than the present wage. Though this does not rule out that incorporating elements of job matching and human capital theory could make the model more realistic, it does point out that there are other factors that have a major influence on the reservation wage.

When specifying the model we made some assumptions concerning the labour market environment that employed individuals face, and one might ask in what sense the results are biased if these assumptions do not hold. First, suppose that forced transitions from the present job into another state of the labour market, say unemployment, occur at a rate s. The method that was used to estimate θ then in reality estimates θ+s. Consequently, θ is over-estimated. It is hard to determine the effect on the estimates of c since the equation for ξ (equation (11)) will change if $s \neq 0$.

FIGURE 3

residual plot for job durations



To examine another assumption, suppose c(0) is different (say smaller) in a next job. In that case future jobs become more advantageous than they are if c were equal for present and future jobs. Consequently, our estimate of c will probably underestimate c that have to be paid when leaving the present job and over-estimate future c. The alternative model specification discussed here can be generalized by assuming that c itself is a job characteristic that is stochastically distributed across jobs and that reveals itself when a job offer arrives. In such a case the set of acceptable job offers is two-dimensional (a job with a low wage is acceptable if c is small enough) and the optimal strategy cannot be represented by a single number. However, by making some assumptions we can obtain a model that closely resembles the model estimated in this chapter. Suppose the job changing costs c can be written as a sum of a non-random term $\alpha.w$ and a random term c_0 that has a distribution function $G(c_0)$ and is independent of the wage being offered. Let $\xi(w|c)$ be the reservation wage that prevails when the present wage is w and the realization of the transaction costs is c. If a number of regularity conditions are satisfied then the optimal strategy can be characterized by the set of functions $\xi(w|c)$ for all c and it can be shown that approximately

$$(14) \qquad \xi(w|c) = w + \frac{\rho+\theta(w)}{1-\alpha\theta(w)} .c$$

with

$$(15) \qquad \theta(w) = \lambda . \int_{-\infty}^{\infty} \int_{\xi(w\,|\,c_0+\alpha w)}^{\overline{w}} dF(x)\ dG(c_0)$$

Of course $\theta(w)$ is again the exit rate out of the present job. Suppose that the observed reservation wage $\tilde{\xi}(w)$ is the expected value over c of $\xi(w|c)$ and suppose that $E(c) = x_1'\gamma_1$. Then taking expectations over c in equation (14) and adding a measurement error results in the equation for $\tilde{\xi}(w)$ in the model from Section 4.3. If we adopt equation (13) as a reduced-form representation of equation (15) then the resulting model is equivalent to the model that is estimated in this chapter. The only consequence of adopting the model with stochastic c is that one should read E(c) instead of c in Tables 3 and 4. Of course the assumption that $\tilde{\xi}(w) = E_c(\xi(w|c))$ is strong and cannot be tested using the data at hand. Alternatively, one might argue that $\tilde{\xi}(w)$ is the minimum over all possible c of $\xi(w|c)$. In that case, if one assumes that the minimum of all possible c can be written as $x_1'\gamma_1$, then again a model can be derived that is equivalent to the model estimated in this chapter.

Another objection to our model specification might be that the costs of moving to another job may depend on the wage in the next job instead of the wage in the present job. In such a case Condition 4 has to be strengthened by substituting $\rho+\lambda$ for λ, in order to guarantee the reservation wage property. It can be proven that if ρ is small then the model is approximately equal to the model we specified before.

4.6. Conclusion

In this chapter we have analyzed the labour market behaviour of employed individuals by estimating a structural on-the-job search model. The model allows for nonzero and wage-dependent costs associated with moving to another job. It was shown that the optimal strategy of an employed individual has the reservation wage property if the costs of moving do not increase too fast as a function of the wage. The model is estimated using Dutch data from 1980–1985, including responses on reservation wages of employed individuals. The results indicate that housing accommodation circumstances, characteristics of the present job and age have a large influence on the willingness to move and on job durations. If one is interested in increasing job mobility then increasing the job offer arrival rate is more effective than decreasing the job changing costs. The estimation results appear to be robust to varying certain assumptions.

There are some straightforward directions for further research. In particular, it may be more realistic to allow for wage rates that vary (stochastically) during the period that one has a job. It also seems interesting to extend the model by allowing jobs to have more than one stochastic characteristic. The presence of such characteristics may bias the estimates of the job changing costs in the model presented. Another topic for further research concerns the quality of the responses to the reservation wage question and the meaning of those responses if other stochastic job characteristics or stochastic job changing costs are present. Note that all the extensions will make the analysis of the optimal strategy and the resulting model equations much more complicated while any empirical analysis will need longitudinal data.

Appendix to Chapter 4

4.A.1. Proof of Proposition 1

It is straightforward but tedious to show that if Conditions 1, 2 and 3a hold then the right–hand side of equation (2) is a mapping T(R) that maps the space of continuous functions on $[0,\bar{w}]$ into itself, and the integrals in (2) are well–defined. Let $C[0,\bar{w}]$ denote the space of continuous functions on $[0,\bar{w}]$. Choose the following norm:

$$\text{for every } X \in C[0,\bar{w}], \quad \|X\| = \sup_{0 \le w \le \bar{w}} |X(w)|$$

Then $C[0,\bar{w}]$ is a Banach space (see Dunford & Schwartz (1957)). We now show that T is a contraction mapping, i.e. that there is an $\alpha \in \langle 0,1 \rangle$ such that for every $R, R^* \in C[0,\bar{w}]$ it holds that $\|T(R)-T(R^*)\| \le \alpha.\|R-R^*\|$. For this it is sufficient that Blackwell's (1965) conditions hold: for every $R,R^* \in C[0,\bar{w}]$:

(i) T *has to be monotone*: *if for every* $0 \le w \le \bar{w}$, $R(w) \le R^*(w)$ *then for every* $0 \le w \le \bar{w}$, $T(R)(w) \le T(R^*)(w)$.

(ii) *There has to be a* $0 \le \beta < 1$ *such that for every constant* δ, $T(R+\delta) = T(R)+\beta.\delta$.

One sees immediately that (i) holds for our mapping T: if $R(x) \le R^*(x)$ and $R(w) \le R^*(w)$ then $\max(R(x)-c(w), R(w)) \le \max(R^*(x)-c(w), R^*(w))$ and $T(R)(w) \le T(R^*)(w)$. To prove (ii) for our T we write

$$\begin{aligned} T(R+\delta)(w) &= \frac{1}{\rho+\lambda}\,(w + \lambda.E_x(\max(R(x)+\delta-c(w), R(w)+\delta))) \\ &= \frac{1}{\rho+\lambda}\,(w + \lambda.E_x(\delta+\max(R(x)-c(w), R(w)))) \\ &= \frac{1}{\rho+\lambda}\,(w + \lambda\delta + \lambda.E_x(\max(R(x)-c(w), R(w)))) \\ &= T(R)(w) + \frac{\lambda}{\rho+\lambda}\,.\delta \end{aligned}$$

Consequently, T is a contraction mapping. From Banach's fixed point theorem (Wouk (1979)) it follows that T which is defined on $C[0,\bar{w}]$ has a unique fixed point. So, from equation (2), R exists and is the unique continuous function on $[0,\bar{w}]$ that solves (2).

4.A.2. Proof of Proposition 2

We know that if Conditions 1, 2 and 3a hold, then there exists a unique solution R(w) of equation (2) which is continuous on $[0,\overline{w}]$. Now it has to be shown that this R(w) is strictly increasing if Condition 4a holds.

Let $0 \leq w < w^* \leq w$. From equation (2) it follows that

$$\text{(A1)} \qquad \rho(R(w^*)-R(w)) = w^*-w$$

$$+ \lambda.E_x(\max(R(x)-c(w^*)-R(w^*), 0) - \max(R(x)-c(w)-R(w), 0))$$

Suppose $c(w^*) \leq c(w)$. If $R(w^*) \leq R(w)$ then the right–hand side (r.h.s.) of (A1) is positive. By contradiction it follows that $R(w^*) > R(w)$. Now suppose $c(w^*) > c(w)$. Equation (A1) can be rewritten as follows,

$$\text{(A2)} \qquad \rho(R(w^*)-R(w)) = w^*-w-\lambda c(w^*)+\lambda c(w)$$

$$+ \lambda.E_x(\max(R(x)-R(w^*), c(w^*)) - \max(R(x)-R(w), c(w)))$$

The first part of the r.h.s. of (A2) is positive due to Condition 4a. Consequently, if $R(w^*) \leq R(w)$ then the r.h.s. of (A2) is positive. Again, by contradiction it follows that $R(w^*) > R(w)$. As a result, for every $0 \leq w < w^* \leq \overline{w}$, $R(w^*) > R(w)$.

4.A.3. Proof of Proposition 3

First it is shown that if Conditions 1, 2, 3b and 4a hold then R(w) has the property that there is a $k>0$ such that

$$\text{(A3)} \qquad \forall\ 0 \leq w < w^* \leq \overline{w},\ R(w^*) \leq R(w) + k.(w^*-w)$$

The function c(w) is continuously differentiable on the closed, bounded interval $[0,\overline{w}]$; therefore there exists an $m>0$ such that, for every $w \in [0,\overline{w}]$, $c'(w) \geq -m$. (Limits in 0 and $\overline{w}$ denote right– and left–hand limits, respectively.) This implies that there is an $m>0$ such that

$$\text{(A4)} \qquad \forall\ 0 \leq w < w^* \leq \overline{w},\quad c(w^*) \geq c(w) - m.(w^*-w)$$

Take without loss of generality $m > \frac{1}{\rho}$. Now it has to be shown that the assertion at the beginning of this proof is true. Suppose it is not true. Then for k=m equation (A3) does not hold, that is, there are w and w^* ($0 \leq w < w^* \leq \overline{w}$) such that $R(w^*) > R(w) + m.(w^* - w)$. Combining this with equation (A4) shows that $R(w^*) + c(w^*) > R(w) + c(w)$. From equation (A1) then, $\rho[R(w^*) - R(w)] \leq w^* - w$. By contradiction it follows that the assertion above is true.

We now proceed to show that R is differentiable. Equation (2) can be rewritten by using the reservation wage property of the optimal strategy.

$$R(w) = \frac{w}{\rho} + \frac{\lambda}{\rho} \cdot \int_{\xi(w)}^{\overline{w}} R(x) - c(w) - R(w) \; dF(x)$$

so

$$\frac{R(w+h) - R(w)}{h} = \frac{1}{\rho} + \frac{\lambda}{\rho h} \cdot \int_{\xi(w)}^{\overline{w}} R(w) + c(w) - R(w+h) - c(w+h) \; dF(x)$$

$$- \frac{\lambda}{\rho h} \cdot \int_{\xi(w)}^{\xi(w+h)} R(x) - c(w+h) - R(w+h) \; dF(x)$$

Therefore,

$$\text{(A5)} \qquad \frac{R(w+h) - R(w)}{h} . \left\{ 1 + \frac{\lambda}{\rho} . \overline{F}(\xi(w)) \right\} - \frac{1}{\rho} + \frac{\lambda}{\rho} . \overline{F}(\xi(w)) . \frac{c(w+h) - c(w)}{h}$$

$$= - \frac{\lambda}{\rho h} \cdot \int_{\xi(w)}^{\xi(w+h)} R(x) - c(w+h) - R(w+h) \; dF(x)$$

Consider the right-hand side of (A5). Assume $h > 0$. If $\xi(w+h) \geq \xi(w)$ then $R(x) \leq c(w+h) + R(w+h)$ which implies that the right-hand side is non-negative. If $\xi(w+h) \leq \xi(w)$ then $R(x) \geq c(w+h) + R(w+h)$ and again the right-hand side is non-negative, so the right-hand side is always non-negative. Further, if $\xi(w+h) \geq \xi(w)$ then $R(x) \geq c(w) + R(w)$ while if $\xi(w+h) \leq \xi(w)$ then $R(x) \leq c(w) + R(w)$. Therefore in both cases the right-hand side is smaller than or equal to

$$\frac{\lambda}{\rho h} \cdot \int_{\xi(w)}^{\xi(w+h)} R(w+h) + c(w+h) - R(w) - c(w) \; dF(x)$$

$$\text{(A6)} \qquad = \frac{\lambda}{\rho} . (\overline{F}(\xi(w)) - \overline{F}(\xi(w+h))) . \left\{ \frac{R(w+h) - R(w)}{h} + \frac{c(w+h) - c(w)}{h} \right\}$$

Suppose $\xi(w) \leq \xi(w+h)$. Because of equation (A3) and Condition 4a, (A6) is smaller than or equal to

$$\text{(A7)} \qquad \frac{\lambda}{\rho}\,.(\overline{F}(\xi(w))-\overline{F}(\xi(w+h)))\ .(k+\frac{1}{\lambda})$$

If, on the other hand, $\xi(w)>\xi(w+h)$ then, because of equation (A4) and because R(w+h) exceeds R(w), (A6) is smaller than or equal to

$$\text{(A8)} \qquad \frac{\lambda}{\rho}\,.(\overline{F}(\xi(w+h))-\overline{F}(\xi(w)))\ .m$$

So the left-hand side of equation (A5) is non-negative and smaller than or equal to the maximum of (A7) and (A8). If we take lim $h\downarrow 0$ of (A7) and (A8) then the result is zero because of the continuity of F and ξ in $\xi(w+h)$ and w+h, respectively. Therefore lim $h\downarrow 0$ of the left-hand side of equation (A5) equals zero. The same holds for lim $h\uparrow 0$. As a result,

$$\text{(A9)} \qquad \lim_{h\to 0}\left[\frac{R(w+h)-R(w)}{h}.(1+\frac{\lambda}{\rho}.\overline{F}(\xi(w))) - \frac{1}{\rho} + \frac{\lambda}{\rho}.\overline{F}(\xi(w)).\frac{c(w+h)-c(w)}{h}\right] = 0$$

Because c is differentiable it follows from (A9) that R is differentiable; R'(w) can be obtained by taking the limit. The functions c', ξ and $\overline{F}$ are continuous in their arguments so from the expression for R'(w) (see equation (4)) if follows that R'(w) is continuous on $[0,\overline{w}]$.

4.A.4. Proof of Proposition 4

If Condition 4b is satisfied then $c'(w).\lambda<1$ and therefore also $c'(w).\lambda\overline{F}(\xi(w)) < 1$. From equation (4) it then follows that $R'(w)>0$ on $[0,\overline{w}]$.

If $0<\xi(w)<\overline{w}$ then equation (3) holds. Let h be small in the sense that also $0<\xi(w+h)<\overline{w}$. Then

$$\text{(A10)} \qquad \frac{R(\xi(w+h))-R(\xi(w))}{\xi(w+h)-\xi(w)}\cdot\frac{\xi(w+h)-\xi(w)}{h} = \frac{R(w+h)-R(w)}{h} + \frac{c(w+h)-c(w)}{h}$$

Because R and c are differentiable the lim $h\to 0$ of the right-hand side of equation (A10) equals R'(w)+c(w). According to the mean value theorem there exists a x(w,h) lying between $\xi(w)$ and $\xi(w+h)$ such that the first part of the left-hand side of equation (A10) equals R'(x(w,h)). Consider the lim $h\to 0$ of R'(x(w,h)). Because x(w,h) lies between $\xi(w)$ and $\xi(w+h)$ and because $\xi(w)$ and $\xi(w+h)$ lie between 0 and $\overline{w}$ it follows that R' is continuous in a neighbourhood of x(w,h). The function ξ is continuous so the lim $h\to 0$ of $\xi(w+h)$ equals $\xi(w)$. This implies that the lim $h\to 0$ of x(w,h) also equals $\xi(w)$. As a result, the lim $h \to 0$ of $R'(x(w,h))$ equals $R'(\xi(w))$ which is positive. Consequently, we can deduce from (A10) that ξ is differentiable in its argument w if $0<\xi(w)<\overline{w}$ and that

$$\text{(A11)} \qquad \xi'(w) = \frac{R'(w)+c'(w)}{R'(\xi(w))}$$

Substitution of equation (4) into equation (A11) gives equation (5). From the continuity of R', ξ and c' their arguments it follows from equation (A11) that $\xi'(w)$ is continuous if $0<\xi(w)<\overline{w}$.

4.A.5. Proof of Proposition 5

The function c(w) is written as $c(w)=\eta+q$, in which q may depend on w but not on η. The reservation wage ξ is expanded as a function of η around $\eta=-q$. Note that η does not depend on w so changing the value of η does not affect w. We use notation $R(w,\eta)$ and $\xi(w,\eta)$ in order to make explicit which value of η holds. It is tedious to show that if η lies in a convex, bounded, closed set (which we take for granted), then R and ξ are continuous both in w and η.

If $\eta=-q$ then $c(w)=0$ and $\xi(w,-q)$ equals w (from the definition of $\xi(w)$). The next thing to do is to calculate $\partial\xi/\partial\eta$ if it exists at $\eta=-q$. We want to compare ξ if $\eta=-q+h$ with ξ if $\eta=-q$. Now for every $w \in <0,\overline{w}>$ one can take h small enough in order to have $0<\xi(w,-q+h)<\overline{w}$. In that case equation (3) is valid,

$$R(\xi(w,-q+h),-q+h) = R(w,-q+h) + h$$

Therefore,

$$\text{(A12)} \qquad \frac{R(\xi(w,-q+h),-q+h) - R(w,-q+h)}{\xi(w,-q+h) - w} \cdot \frac{\xi(w,-q+h) - w}{h} = 1$$

According to the mean value theorem there exists a $x(w,-q+h)$ between w and $\xi(w,-q+h)$ such that the first part of the left-hand side of (A12) equals $R'(x(w,-q+h),-q+h)$ (in which R' denotes the derivative with respect to the first argument) which is positive. From equation (4), this derivative equals

$$\text{(A13)} \qquad \frac{1 - c'(x(w,-q+h)).\lambda\overline{F}(\xi(x(w,-q+h),-q+h))}{\rho + \lambda\overline{F}(\xi(x(w,-q+h),-q+h))}$$

Because of the continuity of ξ in η,

$$\lim_{h \to 0} \xi(w,-q+h) = w$$

and therefore

$$\lim_{h \to 0} x(w,-q+h) = w$$

$$\lim_{h \to 0} \xi(x(w,-q+h),-q+h) = w$$

The lim $h \to 0$ of expression (A13) can now be derived using the continuity of c' and $\overline{F}$. The result is

$$\frac{1-c'(w)\lambda\overline{F}(w)}{\rho+\lambda\overline{F}(w)}$$

From equation (A12), therefore,

$$\text{(A14)} \qquad \lim_{h \to 0} \frac{\xi(w,-q+h) - w}{h} = \frac{\rho+\lambda\overline{F}(w)}{1-c'(w)\lambda\overline{F}(w)}$$

The left-hand side of (A14) is of course $\partial\xi/\partial\eta$ at $\eta=-q$. Note that the denominator of the right-hand side is positive. We now have two terms of the expansion. The remainder is $o(\eta+q)$. Consequently, for arbitrary w,

$$\xi(w,\eta) = w + \frac{\rho+\lambda\overline{F}(w)}{1-c'(w)\lambda\overline{F}(w)} .(\eta+q) + o(\eta+q)$$

which can be rewritten by substituting c(w) for $\eta+q$ and suppressing the dependence of ξ on η,

$$\text{(A15)} \qquad \xi(w) = w + \frac{\rho+\lambda\overline{F}(w)}{1-c'(w)\lambda\overline{F}(w)} .c(w) + o(c(w))$$

For reasons that will be explained in Section 4.3 it is preferable to have $\lambda\overline{F}(\xi(w))$ instead of $\lambda\overline{F}(w)$ in the second term on the right-hand side of (A15). However, the difference between the second term on the right-hand side of (A15) and the second term on the right-hand side of (7) is $o(c(w))$ so equation (7) follows.

4.A.6. The approximation error in a special model

In order to be able to derive explicit results, we have to assume that c does not depend on w. Using equations (2) and (3) one can show that if $c>0$ then the exact $\xi(\overline{w}-\rho c)$ equals $\overline{w}$. Also, from equation (7) it follows that the approximate $\xi(\overline{w}-\rho c)$ equals $\overline{w}$. Consequently, for $w=\overline{w}-\rho c$ the approximation error is zero. Since in general the error is nonzero, this result may suggest that the error (in absolute value) is decreasing in w.

To obtain more results we assume that, in addition to $c'(w)=0$ for all w,

$$F(x) = \frac{e^{\beta(x-w_0)}-1}{e^{\beta(x-w_0)}+\gamma} \qquad\qquad x \geq w_0$$

with $\gamma>-1$, $w_0 \geq 0$ and $\beta>0$, while $F(x)=0$ for $x<w_0$. This wage offer distribution is in conflict with Condition 2 from the main text in the sense that here w_0 may be positive and $\overline{w}=\infty$. However, as noted before, choosing zero to be the lower bound of the wage offers is just a matter of convenience and may be relaxed without loss. Also, we may truncate F at some $\overline{w}>w_0$, but it seems reasonable to suspect that for sufficiently large $\overline{w}$ this will not influence the results. The median of F(x) equals $w_0 + \frac{1}{\beta}.\log(\gamma+2)$ and for $\gamma \leq 1$ the wage offer density decreases on $[w_0,\infty\rangle$. For $\gamma=0$ F(x) reduces to an exponential distribution while for $\gamma<0$ the density decreases at a somewhat faster rate than an exponential density.

We also assume the following relationship between c, ρ, λ, γ and β:

$$\gamma > -\frac{\lambda}{\rho+\lambda}$$

$$c = \frac{1}{\rho\beta}\,.\log\frac{\lambda(\gamma+1)}{\lambda(\gamma+1)+\rho\gamma}$$

These are obviously very strong relationships and it is clear that this model cannot be used for empirical analyses. Not that the relationships imply that c and γ have opposite sign.

In this model there holds that for individuals earning sufficiently high w the probability of obtaining an acceptable offer is negligible. Consequently, the boundary condition of the differential equation (5) is

(A16) $$\lim_{w\to\infty} \xi(w)-w-\rho c = 0$$

Suppose w is such that $w_0<\xi(w)<\infty$. One can show that the exact reservation wage and exit rate out of the present job satisfy

$$\text{(A17)} \qquad \xi(\mathrm{w}) = \mathrm{w} + \rho\mathrm{c} + \frac{1}{\beta}\,.\log(1-\gamma e^{\beta(\mathrm{w}_0-\mathrm{w}-\rho\mathrm{c})})$$

$$\theta(\mathrm{w}) = [\lambda(\gamma+1)+\rho\gamma].e^{\beta(\mathrm{w}_0-\mathrm{w})}$$

By rewriting equation (7) it follows that in this model the approximate $\xi(\mathrm{w})$ is the solution of the following implicit equation,

$$\text{(A18)} \qquad \xi(\mathrm{w}) = \mathrm{w} + \rho\mathrm{c} + \mathrm{c}\ .\frac{\lambda(\gamma+1)}{e^{\beta(\xi(\mathrm{w})-\mathrm{w}_0)}+\gamma}$$

As an example the following parameter values were taken: ρ=λ=0.15, γ=-0.1, w_0=20000, β=0.0001 (time unit: 1 year, money unit: 1 Guilder). Then c=7852, E(x)=29482 and the 95th percentile of F(x) is 48959. For these a priori reasonable values and for w ranging from 20000 to 50000 the exact and approximate $\xi(\mathrm{w})$ were calculated from (A17) and (A18) respectively. The approximation error tends to zero for large values of w. (For instance, if w=50000 then the exact $\xi(\mathrm{w})$ equals 51222 while the approximate $\xi(\mathrm{w})$ equals 51225). This is not surprising since the approximate $\xi(\mathrm{w})$ also satisfies equation (A16). For every w the approximate $\xi(\mathrm{w})$ exceeds the exact $\xi(\mathrm{w})$ and the difference is decreasing in w. However, even for the smallest possible w (20000) the difference is only 82, which is less than 0.4% of the exact $\xi(\mathrm{w})$ (which equals 22029). For w equal to E(x) (29482) the difference is 25, which is less than 0.1% of the exact $\xi(\mathrm{w})$ (which is 30999). Concluding, for parameter values that seem to be reasonable and for a wide range of wage rates the approximation error is quite small.

4.A.7. Proof of relation between γ_1 and γ_2

The theoretical model consists of equations (6), (11) and (12). Further, equation (13) can be interpreted as a summary of $\theta(\mathrm{w})$ in the sense that for an explanatory variable y of c(w), $\gamma_{2\mathrm{y}}.\theta$ which is the partial derivative of θ with respect to y, has to equal

$$\frac{\partial\theta(\mathrm{w})}{\partial \mathrm{c}(0)}\cdot\frac{\partial \mathrm{c}(0)}{\partial \mathrm{y}} = \frac{\partial\theta(\mathrm{w})}{\partial \mathrm{c}(0)}\cdot\gamma_{1\mathrm{y}}$$

in which $\partial\theta(\mathrm{w})/\partial\mathrm{c}(0)$ can be deduced from equations (6), (11) and (12). It follows that $\gamma_{1\mathrm{y}}$ and $\gamma_{2\mathrm{y}}$ have opposite signs if $\partial\theta(\mathrm{w})/\partial\mathrm{c}(0)<0$. From equation (6) $\partial\theta(\mathrm{w})/\partial\mathrm{c}(0)$ has the opposite sign of $\partial\xi(\mathrm{w})/\partial\mathrm{c}(0)$. The latter can be written as

$$\frac{\partial\xi(w)}{\partial c(0)} = \frac{(\rho+\theta)(1-\alpha\theta)}{(1-\alpha\theta)^2 + \lambda f(\xi)\,.\,(1+\alpha\rho).c(w)}$$

with f=F'. From equation (6),

$$-\lambda f(\xi) = \frac{\theta'(w)}{\xi'(w)}$$

Consequently,

$$\frac{\partial\xi(w)}{\partial c(0)} = \frac{(\rho+\theta)\,.\,\xi'(w)}{1+\alpha\rho}$$

and the result follows.

4.A.8. Derivation of elasticities

The general procedure is as follows. From the theoretical specification (equations (6), (11) and (12)) that underlies the empirical specification ((11), (12) and (13)) we derive expressions for the derivatives of θ and ξ with respect to c(w) and λ. These expressions contain the unknown λ and F' but by using the equality of (6) and (13) and the equality of the derivatives of (6) and (13) with respect to w we can make some substitutions that result in calculable functions. Consider the elasticities with respect to c(w). From (6), (11) and (12),

(A19) $$\frac{\partial\theta}{\partial c(w)} = -\,\lambda f(\xi)\cdot\frac{\partial\xi}{\partial c(w)}$$

(A20) $$\frac{\partial\xi}{\partial c(w)} = \frac{1+\alpha\rho}{(1-\alpha\theta)^2}.c(w).\frac{\partial\theta}{\partial c(w)} + \frac{\rho+\theta}{1-\alpha\theta}$$

From (A19) and (A20),

(A21) $$\frac{\partial\theta}{\partial c(w)} = \frac{-\lambda f(\xi).(\rho+\theta)(1-\alpha\theta)}{(1-\alpha\theta)^2 + \lambda f(\xi).(1+\alpha\rho).c(w)}$$

By differentiating equations (6) and (13) with respect to w,

$$\lambda f(\xi) = \frac{\beta_1}{w}\cdot\frac{\theta}{\xi'(w)}$$

Now $\xi'(w)$ can be obtained simply by differentiating (11) after substitution of

(12) and (13). As a result (A21) can be simplified to

$$\frac{\partial\theta}{\partial c(w)} = -\frac{(\rho+\theta)}{(1+\alpha\rho)} \cdot \frac{\beta_1\theta}{w}$$

Analogously one can derive for example that

$$\frac{\partial \log \theta}{\partial \log \lambda} = 1 - \frac{\beta_1 . \theta . c(w)}{w(1-\alpha\theta)}$$

etc. The key identifying restriction is the equality between (6) and (13). In effect, the functional form of (13) implicitly ties up the hazard of the wage offer distribution on the relevant wage interval. This is not enough to identify F and λ but it is enough to identify elasticities with respect to λ and c(w).

CHAPTER 5

THE EFFECT OF AN INCREASE OF THE RATE OF ARRIVAL OF JOB OFFERS ON THE DURATION OF UNEMPLOYMENT

5.1. Introduction

In this chapter we examine the effect of an increase of the job offer arrival rate on the hazard of the unemployment duration distribution in job search models. It is shown that previously derived sufficient conditions on the wage offer distribution for this effect to be positive can be generalized considerably at no cost. This has some important implications for both structural and reduced-form empirical analyses of unemployment duration.

In the empirical analysis of unemployment duration one of the main issues concerns the influence of the rate of arrival of job offers on duration. The choice of exogenous variables in reduced-form duration models is, among other things, governed by the belief that these variables may characterize individual differences in this arrival rate (see for example Lancaster (1979) and Ham & Rea (1987)). The elasticity of the expected duration with respect to the arrival rate is one of the parameters of interest in the analysis of structural job search models (see for example Lancaster & Chesher (1983)). Further, the influence of the arrival rate on duration is of some importance for the macro implications of search theory (see for example Vroman (1985)).

It is well known that changing the job offer arrival rate has two opposite effects on the hazard or exit rate out of unemployment and therefore on the expected duration of unemployment. First, there is a positive effect on the hazard because of the increased expected number of occasions at which one is able to leave unemployment. Secondly, there is a negative effect because of the increased selectivity of the searcher in face of this increased opportunity to leave unemployment. In job search models the sign and magnitude of the net effect depend on other variables affecting the optimal strategy of an unemployed individual (like the wage offer distribution and the subjective rate of discount) and therefore the sign of the net effect is ambiguous (see for example Flinn & Heckman (1982) and Mortensen (1986)). Consequently, it seems that from the estimates of a reduced-form duration model one cannot conclude anything about the sign of the relationship between the job offer arrival rate and the covariates in the reduced-form hazard function. Also, it seems that such estimates cannot be used to check whether the data are in agreement with a priori beliefs about e.g. the sign of the relationship between the local rate of unemployment and the expected duration of unemployment if it is believed that this relationship mainly acts by way of the arrival rate.

Recently, a number of papers have been written in which sufficient conditions are derived for the net effect on the hazard to be non-negative in a basic job search model. (In Section 5.2 a small survey is presented.) These conditions are stated in terms of the shape of the wage offer distribution. However, they are not satisfied for most families of distributions generally used to model wage offer distributions in structural job search models and other income-related distributions. Therefore they seem to be of limited practical interest as a guide to the interpretation of reduced-form model estimates. Also, since the assumed families of wage offer distributions in structural empirical analyses generally do not satisfy those conditions, the suspicion may arise that the estimates of structural job search models are sensitive with respect to the assumed family. If a slight change of the shape of the wage offer distribution implies a sign-reversal of the relationship between the hazard and the job offer arrival rate, then a very small misspecification of the family of wage offer distributions can have dramatic consequences for the quality of the estimation results.

In Section 3 of this chapter it is shown that the sufficient conditions on the wage offer distribution that guarantee that the relationship between the hazard and the arrival rate is non-negative can be weakened at no cost, to include virtually every conceivable (wage offer) distribution. We present the generalized counterparts of the conditions derived before and examine for which of the well-known families of distributions they are satisfied. In fact, it appears that in a certain sense the class of distributions satisfying the generalized conditions is almost equivalent to the class of non-defective distributions.

In Section 5.4 we show that the generalized conditions also imply another regularity in the relationship between the hazard and its explanatory variables, namely that the absolute value of the elasticity of the hazard with respect to the level of unemployment benefits is increasing in the latter. Section 5.5 deals with extensions of the basic job search model framework that is generally used to analyze the relationship between the hazard and the arrival rate. We show that for a wide class of utility functions the results in Section 5.3 remain valid if the individuals are assumed to maximize expected utility instead of income. Often, negative duration dependence of the hazard is attributed to a job offer arrival rate that is a decreasing function of duration. Therefore, in Section 5.5 we also pay some attention to the consequences of changes in the arrival rate during a given spell of unemployment, i.e. to non–equilibrium changes generating nonstationarity of the optimal strategy of an unemployed individual. It appears that the analysis of the nonstationary model can be linked to the analysis of the stationary model in Section 5.3. Section 5.6 concludes and lists the benefits of the analysis for empirical work using either reduced–form duration models or structural search models.

5.2. The model

5.2.1. *Job search theory and model specification*

In this subsection we present the basic job search model generally used to analyze the relationship between the job offer arrival rate and the duration of unemployment. The next subsection surveys the results obtained thus far.

Job search theory tries to describe the behaviour of unemployed individuals in a dynamic and uncertain world. Job offers arrive according to a Poisson process with arrival rate λ. Such job offers are random drawings (without recall) from a wage offer distribution with distribution function $F(w)$. Every time an offer arrives the decision has to be made whether to accept the offer or to reject it and search further. Once a job is accepted it will be held forever at the same wage. It is assumed that individuals know λ and $F(w)$ but that they do not know in advance when job offers arrive and what wages are associated with them. During the spell of unemployment a benefit b is received. Unemployed individuals aim at maximization of their own expected present value of income (over an infinite horizon). The subjective rate of discount is denoted by ρ. The variables λ, w, b and ρ are measured per unit time period. It is assumed that the model is stationary. This means that λ, $F(w)$, b and ρ are assumed to be independent of unemployment duration and calendar time and independent of all events during unemployment.

For a precise analysis of the relationship between the job offer arrival rate and unemployment duration it is necessary to state explicitly the assumptions that the variables λ, $F(w)$, b and ρ have to satisfy. The following weak assumptions ensure that attention is restricted to economically meaningful cases and guarantee the existence of the optimal strategy.

1. $F(w)$ *is a distribution function of* w *that is continuous on* $<-\infty,\infty>$. *There*

is an interval $<\alpha,\beta>$ *with* $0\leq\alpha<\beta\leq\infty$ *such that* $F(\alpha)=0$, $\lim_{w\to\beta} F(w)=1$ *and* $F(w)$ *is twice differentiable on* $<\alpha,\beta>$, *its first derivative* $f(w)$ *being positive on* $<\alpha,\beta>$ *and its second derivative being continuous on* $<\alpha,\beta>$. *The mean of the distribution of* w *is finite.*

2. $0<\lambda<\infty$, $0<\rho<\infty$, $0\leq b<\infty$.

Assumption 1 rules out that there are wage offers that have a positive probability of occurrence. On the other hand it is allowed that the wage offer density $f(w)$ is discontinuous or even non-existent at the boundary points α and β of the interval of support. For simplicity we then define $f(\alpha)=0$ and $f(\beta)=0$, respectively. We restrict attention to positive λ since a zero λ implies an infinite duration of unemployment. If $\rho=0$ or $E(w)=\infty$ then the expected present value of income does not exist. The condition that the level of unemployment benefits is non-negative deserves some extra attention. Previous papers on the relationship between λ and the duration of unemployment have not stressed this condition since b can be interpreted as the official unemployment benefits level minus per-period search costs, and the difference of two positive variables has an indeterminate sign. However, in order to survive individuals need a positive net (of search costs) income flow. Of course assets and capital markets may be available for individuals for which the difference between benefits and search costs is negative, but then searching implies dissaving, which in turn implies nonstationarity of the optimal strategy (see Danforth (1979): a model in which income minus search costs is negative cannot be stationary). Casual empiricism suggests that, at least in Western European countries, search costs (noticing advertisements when reading newspapers, contacting potential employers, making expenses-paid visits to them etc.) are small relative to unemployment benefits. Indeed, job search may be highly subsidized by means of government-paid job-search training facilities and institutionalized contacts between employers and job searchers. Finally, we might note that in structural empirical analyses b is often taken to be identical to the received amount of unemployment benefits (see for example Lancaster & Chesher (1983), Lynch (1983), Narendranathan & Nickell (1985), Ridder & Gorter (1986), van den Berg (1990c)).

For the job search model described above (and satisfying Assumptions 1 and 2) it has been shown many times that the optimal strategy can be characterized by a reservation wage (see e.g. Lancaster & Chesher (1983)). A job offer is acceptable if its wage exceeds the reservation wage ϕ while a wage below ϕ induces one to reject the offer and search for a better one. The reservation wage is the unique solution to

(1) $$\phi = b + \frac{\lambda}{\rho}\cdot\int_{\phi}^{\infty}(w-\phi)dF(w)$$

From Assumptions 1 and 2 it follows that $b<\phi<\infty$ which implies that ϕ is always positive. The hazard (or exit rate out of unemployment, or transition rate from unemployment into employment) θ can be written as the product of the job offer arrival rate and the conditional probability of accepting a job offer.

(2) $$\theta = \lambda\overline{F}(\phi) \qquad\qquad \overline{F}=1-F$$

Because of the stationarity assumption, θ does not depend on the elapsed duration of unemployment. Consequently, the duration of unemployment t has an exponential distribution with parameter θ, so $E(t)=1/\theta$.

The objective is to examine the effect of a change of the job offer arrival rate λ on the duration of unemployment, or, more specific, to examine the sign of $\partial E(t)/\partial\lambda$. Of course this is equivalent to examining the sign of $\partial\theta/\partial\lambda$. Knowledge of this sign allows one to rank the hazards (and the expected unemployment durations) of individuals who differ with respect to λ but are identical with respect to the other explanatory variables in the model. Alternatively, the sign of $\partial\theta/\partial\lambda$ describes the long-run (or, equilibrium) effect of an anticipated change of λ for a particular unemployed individual, in the sense that it shows whether the θ prevailing infinitely long before a change of λ is larger or smaller than the θ prevailing after the change of λ. (This follows from Chapter 3. In Subsection 5.5.2 we examine in detail the time path of θ in case of anticipated changes of λ.) It is sometimes argued that $\partial\theta/\partial\lambda$ can also be used to describe the effect of unanticipated changes of λ (see e.g. Burdett (1981)). However, it seems plausible to assume that if such changes can occur, then individuals are aware this. In that case the model should incorporate the individual's subjective assessments of the probability that various changes will occur.

If $\phi<\alpha$ or $\phi>\beta$ then examining the sign of $\partial\theta/\partial\lambda$ is trivial since then $\theta=\lambda$ and $\theta=0$, respectively. In the sequel therefore attention is restricted to cases in which $\alpha<\phi<\beta$ (if $\phi=\alpha$ or $\phi=\beta$ then $\partial\theta/\partial\lambda$ may not exist).

5.2.2. *Previous results*

Many papers have been written presenting sufficient conditions for $\partial\theta/\partial\lambda$ to be non-negative. All these conditions are stated in terms of the shape of the wage offer distribution. Before presenting them we define some functions. The expectation of w conditional on w>x as a function of x is denoted by $\mu(x)$, so $\mu(x)=E_W(w|w>x)$. Of course $\mu(\phi)$ equals the expected wage in employment for an unemployed individual with reservation wage ϕ.

We define Q(x) by

$$(3) \qquad Q(x) = \int_x^\infty (w-x)dF(w)$$

which, by virtue of $E_W(w)<\infty$, can be rewritten as

$$\int_x^\infty \overline{F}(w)dw$$

From the definition of $\mu(x)$ it follows that $Q(x)=\overline{F}(x).(\mu(x)-x)$. $Q(\phi)$ equals the expected excess income flow of an individual with reservation wage ϕ at the moment that a job is being offered.

Yet another function, $\psi(x)$, is defined by

$$(4) \qquad \psi(x) = \frac{f(x)}{\overline{F}(x)} \qquad\qquad x<\beta$$

This is the hazard or failure rate associated with the distribution F. For small dx the expression $\psi(x)dx$ can be interpreted as the probability that a wage offer is in the interval $[x,x+dx>$ if it is given that this wage offer exceeds x. In order to avoid confusion with the hazard θ associated with the duration distribution, we will call ψ the failure rate of F. Note that μ, Q and ψ are well-defined.

A function g(x) is called log concave if there is an interval I such that log g(x) is concave on I and g(x) is positive on I but vanishes exterior to I. This is equivalent to saying that g(x) is a Pólya frequency function of order 2 (see Karlin (1968) or Karlin (1982)). If g(x) is twice differentiable on I then log concavity can be checked by examining the sign of $g''(x).g(x)-(g'(x))^2$ for every $x\in I$.

Before surveying the papers in which sufficient conditions were derived for $\partial\theta/\partial\lambda$ to be non-negative, we first list these conditions.

1a. $\mu'(x)\leq 1$ *at* $x=\phi$.
1b. log Q(x) *is a concave function of* x *at* $x=\phi$.
2a. $\psi'(x)\geq 0$ *for every* $x\in<\alpha,\beta>$.
2b. $\overline{F}(x)$ *is a log concave function of* x.

3a. f(x) *is a log concave function of* x.

One can show that 1a $\Leftrightarrow$ 1b, 2a $\Leftrightarrow$ 2b and 3a $\Rightarrow$ 2a $\Rightarrow$ 1a. Conditions 2a, 2b and 3a do not make any reference to the actual ϕ; hence, if one of these conditions holds then $\partial\theta/\partial\lambda \geq 0$ for all admissible values of b, λ and ρ. Condition 1a states that if the reservation wage slightly increases, then the expected wage in employment increases by a smaller amount. Equivalently, the expected wage in employment in excess of the reservation wage $E_W(w-\phi|w>\phi)$ is decreasing in the reservation wage. If Condition 2a holds then the probability of obtaining a wage offer $w\in[x,x+dx>$ with dx small, conditional on w>x, increases as x increases. In terms of the literature on reliability, Condition 2a states that the wage offer distribution F has the IFR (increasing failure rate) property. Also, if Condition 1a holds for every $\phi\in<\alpha,\beta>$ then F has the DMRL (decreasing mean residual life) property (see Hollander & Proschan (1984)).

Burdett (1981) proved that 1a $\Rightarrow$ $\partial\theta/\partial\lambda \geq 0$. By slightly modifying a result in Goldberger (1983) he also showed that 3a $\Rightarrow$ 1a. These results were re-published in Burdett & Ondrich (1983). Both Burdett (1981) and Burdett & Ondrich (1983) presented a proposition which states that if Condition 1a does not hold for any ϕ, that is, if F is such that $\mu'(x)>1$ for every $x\in<\alpha,\beta>$, then there are always positive values of ρ such that $\partial\theta/\partial\lambda<0$. However, this is incorrect, as will be shown in Section 3 of this chapter. Sattinger (1985) incorrectly cites Burdett & Ondrich (1983) by stating that if Condition 3a does not apply, then $\mu'(x)>1$ for all x. A counter-example for this statement is provided by the distribution with $\overline{F}(x)=\exp(-\frac{1}{3}x^3-x)$ for every $x\geq 0$: it can be checked easily that the log of the density f(x) is not concave for small positive x; nevertheless $\overline{F}(x)$ satisfies Condition 2b and therefore the Condition 1a is also satisfied.

Flinn & Heckman (1983) proved that 1b $\Rightarrow$ $\partial\theta/\partial\lambda \geq 0$ and, by using a result in Barlow & Proschan (1975), it was shown that 3a $\Rightarrow$ 2b $\Rightarrow$ 1b. Vroman (1985) also proved that 1b $\Rightarrow$ $\partial\theta/\partial\lambda \geq 0$ and, using a theorem in Karlin (1968), that 3a $\Rightarrow$ 2b $\Rightarrow$ 1b. Further, the equivalence of 1a and 1b is established and it is suggested that if Condition 3a is not satisfied then $\partial\theta/\partial\lambda<0$ is likely to occur only if ϕ is large. As will be shown in Section 5.3 this suggestion is questionable. Finally, Jensen & Vishwanath (1985) proved that 2a $\Rightarrow$ 1a $\Rightarrow$ $\partial\theta/\partial\lambda \geq 0$. It should be noted that some of the papers mentioned use a discrete-time model while others use a continuous-time model. However, for the sign of $\partial\theta/\partial\lambda$ this makes no difference. (If time is discrete then λ and θ are probabilities rather than rates.) Also, most of the papers mentioned do not explicitly state assumptions (like Assumptions 1 and 2) on the values of λ, F(w), b and ρ.

Generally, Conditions 2a and 3a are the easiest to check for a given wage offer distribution. The class of distributions satisfying Conditions 2a and 3a includes the exponential and beta families, the families of logistic and extreme value distributions that are truncated from below at zero and the family of uniform distributions for which the lower point of support is non-negative. (Recall that we are only interested in distributions satisfying Assumption 1, which implies among other things that the mean has to be finite and that the interval $<-\infty,0]$ must have zero probability. See Mood, Graybill & Boes (1974) for a list of the parameterized F and f associated with the families of distributions mentioned.) Contrary to what is stated in Flinn & Heckman (1983), the class of distributions satisfying Conditions 2a and 3a also includes the whole family of normal distributions truncated at zero. Further, it includes the members of the Weibull family ($\overline{F}(x) = \exp(-\beta x^{\alpha})$ for $x>0$ with $\beta>0$ and $\alpha>0$) that have $\alpha \geq 1$ and the members of the gamma family ($f(x) \sim x^{\gamma}.e^{-\lambda x}$ for $x>0$ with $\lambda>0$ and $\gamma>-1$, note that here λ denotes a parameter of f and not the job offer arrival rate) that have $\gamma \geq 0$. More examples of distributions that have a log concave density are in Karlin (1982). It is easily seen that truncation from below does not invalidate the conditions. Further, convolutions of distributions satisfying Conditions 2a or 3a also satisfy those conditions (see Barlow & Proschan (1975) and Karlin (1982)).

In a sense the family of exponential distributions (possibly truncated from below) is a boundary case for all conditions, since for these distributions the conditional mean does not depend on the point of truncation ($\mu'(x)=1$ for $x>\alpha$), the failure rate is constant ($\psi'(x)=0$ for $x>\alpha$) and the log of the density is linear on the interval of support. All distributions that have a failure rate that strictly decreases on some interval do not satisfy Conditions 2a and 3a. Moreover, if the log of the density is strictly convex on $[\phi,\infty>$ or if the failure rate strictly decreases on $[\phi,\beta>$ (the former implies the latter) then Condition 1a is not satisfied either. Therefore in such cases the conditions derived before cannot be used to check whether $\partial\theta/\partial\lambda \geq 0$. This would not be much of a problem if it could be safely assumed that wage offer distributions have the IFR property. However, the families of distributions that are widely believed to be able to represent wage offer distributions and other income-related distributions and which are typically used to model wage offer distributions in structural empirical job search analyses all have a failure rate that strictly decreases on some interval. (This will be shown in the next paragraph.) Indeed, one can provide a priori plausible theoretical reasons for income-related distributions to have a failure rate that decreases on some interval (see e.g. Singh & Maddala (1976): the basic idea is that the ability to make more money may well increase with one's income, which would imply that the expected residual income increases with the point at which the income distribution is truncated from below, and therefore that this distribution cannot have the IFR property) and there is much empirical evidence that confirms that such distributions have a failure rate that decreases after a certain point (see e.g. Singh & Maddala (1976) and McDonald (1984)). Barlow & Proschan (1975) and Karlin (1968) give other illustrations of the limitations of Conditions 2a and 3a.

The most popular families of distributions used to model wage offer distributions in a structural empirical search framework are the log-normal family and the Pareto family. (One can give theoretical foundations for their use as models of income-related distributions, see e.g. Cramer (1971).) For example Narendranathan & Nickell (1985), Blau & Robins (1986b), Wolpin (1987) and van den Berg (1990c) use the log-normal family while Lancaster & Chesher (1983), Lancaster (1985), Ridder & Gorter (1986) and Jones (1988) use the Pareto family. The distributions of the log-normal family have a failure rate that strictly decreases for sufficiently large values (see e.g. Lancaster (1990)). The family of Pareto distributions has a strictly decreasing failure rate on the whole interval of support (contrary to what is stated in Pratt (1981)).

The family of Singh-Maddala distributions ($\overline{F}(x)=(1+a_1x^{a_2})^{-a_3}$ for $x>0$ with $a_1,a_2,a_3>0$; if $a_2.a_3>1$ then the mean is finite) is well known for its good empirical performance as a model for income distributions (see McDonald (1984)). Stern (1988) uses this family to model wage offer distributions in a search theoretic framework. In this family the failure rate strictly decreases for sufficiently large values; in fact for some parameter values it strictly decreases on the whole interval of support. The family of Singh-Maddala distributions contains as a special case the family of log-logistic distributions (see Kalbfleisch & Prentice (1980)). The distributions of the log-uniform family ($\overline{F}(x) = (\log \beta - \log x)/(\log \beta - \log \alpha)$ for $\alpha<x<\beta$ with $0<\alpha<\beta<\infty$), that is used by van den Berg (1990b) to model wage offer distributions in a structural search model, have a failure rate that strictly decreases for sufficiently small values if $1 + \log \alpha < \log \beta$.

Other families of which the distributions fail to satisfy Condition 2a are the family of t distributions truncated from below at zero and the family of F distributions, since these distributions have a density of which the log is strictly convex for sufficiently large values. (It is strictly convex on the whole interval of support for F distributions with 'degrees of freedom for the numerator' equal to 1 or 2.) Pratt (1981) states that all t distributions (and therefore all t distributions truncated from below at zero) do satisfy Condition 2b. Clearly, this is incorrect. (In fact, this can be checked easily by examining a table of the distribution functions.) Note that again attention is restricted to distributions satisfying Assumption 1, i.e. that have a finite mean etc. The members of the gamma family for which $-1<\gamma<0$ and the members of the Weibull family for which $0<\alpha<1$ have a failure rate that strictly decreases on the whole interval of support.

The discussion above on the restrictiveness of the conditions derived before has some important implications. First of all, the conditions derived before are of no practical interest as a guide for the interpretation of the estimates of reduced-form models of unemployment duration. Clearly it cannot be ruled out that the wage offer distribution is such that neither of the conditions is satisfied, so if an explanatory variable (e.g. number of working individuals in the household) is found to have a positive influence on the hazard and this variable is believed not to influence F or ρ, then one cannot conclude that it has a positive influence on λ. Also, if it is believed that the influence of an explanatory variable (e.g. local unemployment percentage) on the hazard mainly acts by way of λ then reduced-form estimates cannot be used to check whether the data are in agreement with these prior beliefs.

Another implication concerns the estimation of structural job search models. Since the assumed families of wage offer distributions in structural empirical analyses generally do not satisfy the conditions derived before, the suspicion may arise that the estimates of parameters in such models are sensitive with respect to the assumed family. If a slight change of the shape of the wage offer distribution implies a sign-reversal of the relationship between the hazard and λ, then a very small misspecification of the family of wage offer distributions can have dramatic consequences for the quality of the estimation results.

In the next section it is shown that the scope for the somewhat negative implications that seem to result from the analysis of the conditions derived before can be narrowed a great deal.

5.3. Weak new conditions on the wage offer distribution

5.3.1. *Generalizations of the previous results*

In this subsection we state the new results on the sign of $\partial\theta/\partial\lambda$ and show that they are more general than previous results. The sufficient conditions for $\partial\theta/\partial\lambda \geq 0$ derived before can be replaced by weaker versions. We show that all families of distributions that were shown in the previous section not to satisfy the conditions derived before, do satisfy the new conditions.

Proposition 1.

In a job search model that satisfies Assumptions 1 *and* 2, *and in which* $\alpha<\phi<\beta$, *sufficient conditions for* $\partial\theta/\partial\lambda$ *to be non-negative are*

Ia. $\dfrac{d \log \mu(x)}{d \log x} \leq 1$ *at* $x=\phi$.

Ib. $\log Q(e^{x})$ *is a concave function of* x *at* $x = \log \phi$.

IIa. $\dfrac{d \log \psi(x)}{d \log x} \geq -1$ *for every* $x \in \langle\alpha,\beta\rangle$.

IIb. $\bar{F}(e^{x})$ *is a log concave function of* x.

IIIa. $f(e^{x})$ *is a log concave function of* x.

There holds that Ia $\Leftrightarrow$ Ib, IIa $\Leftrightarrow$ IIb and IIIa $\Rightarrow$ IIa $\Rightarrow$ Ia. *Also,* Ia *and* IIa *are weaker than* 1a *and* 2a, *respectively. If for every* $x \in \langle\alpha,\beta\rangle$ $f'(x) \leq 0$ *then* IIIa *is weaker than* 3a.

The proof is in the appendix. Conditions IIa, IIb and IIIa do not make any reference to the actual ϕ; hence, if one of these conditions holds then $\partial\theta/\partial\lambda \geq 0$ for all admissible values of b, λ and ρ.

Condition Ia is necessary and sufficient for $\partial\theta/\partial\lambda \geq 0$ if and only if b=0 (see the appendix). This suggests a necessary and sufficient condition for the general case. It is well known that a number of properties of ϕ and θ as solutions of equations (1) and (2) are invariant under simultaneous and equally large shifts of b and F (by which we mean that F is translated by an amount equal to the change of b, and in the same direction). In particular, one can show that $\partial\theta/\partial\lambda$ is invariant in this sense. Consequently, necessary and sufficient for $\partial\theta/\partial\lambda \geq 0$ is that Condition Ia holds for the ϕ (as solution of equation (1)) and F that are obtained by shifting b and F in the sense above such that the new value of b equals zero. Note that Condition Ia itself is not invariant under these shifts. Also note that the fact that some points of support of the new F may be negative raises no problem since we are only performing a mathematical trick here. Nevertheless, the necessary and sufficient condition derived here is not very transparent, nor is it convenient to work with, since it critically depends on the value of b. Therefore, in the sequel, we restrict attention to the conditions presented in Proposition 1.

One may say somewhat loosely that Condition Ia states that if the reservation wage ϕ slightly changes, then the proportional change of the expected wage in employment does not exceed the proportional change of ϕ. For instance, a 10% increase of the reservation wage implies an increase of the expected wage in employment of at most 10%. The expected wage in employment by definition exceeds the reservation wage, so it is obvious that Condition Ia is weaker than Condition 1a.

Because of the intimate relationship between the functions μ' and f (see the appendix) one may interpret Condition Ia as being a restriction on the magnitude of f at ϕ, relative to the magnitude of f at values larger than ϕ. If for most $x>\phi$ $f(x)$ is much smaller than $f(\phi)$, then the proportion of acceptable job offers $\overline{F}(\phi)$ decreases very much when ϕ increases due to an increase of λ, thus offsetting the direct effect of this increase of λ on θ. Note that this interpretation of Condition Ia is very loose, since it does not deal with the precise relationship between μ' and f, nor with the magnitude of the increase of ϕ due to an increase of λ. Also note that Condition 1a can be interpreted in the same way, though of course in that case the restriction on $f(\phi)$ is stronger. The functions μ and μ' at ϕ can be expressed in terms of the function $\psi(x)$ on $[\phi,\beta>$, so it is not surprising that a sufficient condition for Condition Ia can be expressed in terms of $\psi(x)$ on $[\phi,\beta>$. Condition IIa extends this to $\psi(x)$ on $<\alpha,\beta>$. Note that because Condition IIa allows the conditional probability that a wage w in $[x,x+dx>$ (with dx small) is offered to decrease in x, it is clear that Condition IIa is weaker than Condition 2a. The only remaining restriction in Condition IIa is that this probability may not decrease more than proportional in response to an increase of x.

The conditions in Proposition 1 are stated in a form that highlights the differences from the conditions derived before. We now present a number of alternative formulations of these conditions. It is easily shown that Condition Ia is equivalent to the condition that

$$(5) \qquad \frac{d\ \log(\mu(x)-x)}{d\ \log x} \le 1 \text{ at } x=\phi$$

(this will be used in Subsection 5.3.3) while Condition IIa is equivalent to the condition that $x.\psi(x)$ is non-decreasing on $<\alpha,\beta>$. A distribution satisfying the latter is said to have the IPFR (increasing proportionate failure rate) property (see Singh & Maddala (1976)). Conditions IIa, IIb and

IIIa can be characterized in terms of the distribution of log wage offers z. To distinguish the functions F, ψ and f corresponding to the wage offer distribution on the one hand and similar functions for the distribution of log wage offers on the other, we use subscripts w and z. Because $\overline{F}_z(x) = \overline{F}_w(e^x)$, $f_z(x) = e^x.f_w(e^x)$ and $\psi_z(x) = e^x.\psi_w(e^x)$, it follows that Conditions IIb, IIIa and IIa are equivalent to the conditions that $\overline{F}_z(x)$ is log concave, $f_z(x)$ is log concave, and $\psi'_z(x) \geq 0$ for every $x \in \langle \log\alpha, \log\beta \rangle$, respectively. In other words, Conditions IIa, IIb and IIIa can be checked by examining Conditions 2a, 2b and 3a for the distribution of the log wage offers. Note that if $\alpha<1$ then $F_z(0)>0$, so the distribution of z need not satisfy the properties of the distribution of w as stated in Assumption 1. The results in this paragraph enable one to give characterizations of the classes of distributions satisfying Conditions IIa or IIIa by using characterizations (like those in Dharmadhikari & Joag-dev (1988)) of the classes of distributions with the IFR property or with a log concave density.

We now examine which families of distributions satisfy Condition IIa. Because this condition is weaker than the Conditions 2a and 3a, all distributions listed in Section 5.2 that satisfy the latter conditions also satisfy Condition IIa. If w has a log-normal distribution, then z = log w has a normal distribution. Since all normal distributions have the IFR property, it follows that the family of log-normal distributions satisfies Condition IIa. Analogously one can show that the log-uniform family satisfies Condition IIa. The Pareto family ($\overline{F}(x)=(w_0/x)^{\nu}$ for $x>w_0$ with $w_0>0$ and $\nu>1$) has $\psi(x)=\nu/x$ for $x>w_0$ so $x.\psi(x)$ does not decrease on $\langle w_0,\infty \rangle$ and Condition IIa is satisfied. In a sense, the Pareto family is a boundary case for all the conditions listed in Proposition 1, because for all distributions in this family $d\log\mu(x)/d\log x = 1$ for $x>w_0$, $d\log\psi(x)/d\log x = -1$ for $x>w_0$, and $\log f(e^x)$ is linear on $\langle w_0,\infty \rangle$. The results on the Pareto family can also be deduced from the fact that z = log w has a shifted exponential distribution if w has a Pareto distribution. The Singh-Maddala family of distributions with parameters a_1,a_2 and a_3 (see Section 5.2) has $\psi(x)=a_1a_2a_3x^{a_2-1}/(1+a_1x^{a_2})$ for $x>0$ and it is readily shown that this family satisfies Condition IIa. As a result, all families of distributions that (i) are widely believed to be able to represent wage offer distributions and other income-related distributions and that (ii) are typically used to model wage offer distributions in structural empirical job search analyses and that (iii) do not satisfy the conditions derived before for $\partial\theta/\partial\lambda$ to be non-negative, do satisfy Condition IIa. This implies that if the wage offer distribution in the job search model belongs to one of these families, then $\partial\theta/\partial\lambda \geq 0$ for all possible values of the explanatory variables in the model. From an empirical point of view, Singh & Maddala (1976) argue that to model income-related distributions one should use a family of distributions that have the IPFR property, that is, distributions that satisfy Condition IIa. Moreover, in the next subsection it is shown that in a certain sense the class of distributions satisfying Condition IIa is almost equivalent to the class of non-defective distributions. Clearly, these results narrow the scope for the problems mentioned at the end of Section 5.2.

The other families of distributions that were shown in Section 5.2 not to satisfy the conditions derived before also all satisfy Condition IIa. The family of t distributions truncated from below at zero and the family of F distributions have a density satisfying Condition IIIa. The members of the Weibull family for which $0<\alpha<1$ have $x.\psi(x)=\alpha\beta.x^{\alpha}$ which does not decrease. The members of the gamma family for which $-1<\gamma<0$ have a density satisfying Condition IIIa.

It is easily seen that truncation of the wage offer distribution from below does not invalidate the conditions listed in Proposition 1. Although Condition IIIa is not affected by truncation from above, the other conditions are. In fact, any distribution not satisfying those conditions can be made to satisfy them by truncating it from above at a sufficiently small value. Alternatively, if F is a distribution that is truncated from above then $\partial\theta/\partial\lambda\geq 0$ for sufficiently large values of ϕ. The appendix contains a more detailed discussion of these issues.

Burdett (1981) and Burdett & Ondrich (1983) state that if $\mu'(x)>1$ for every $x\in<\alpha,\beta>$ then there is always a positive value of ρ such that $\partial\theta/\partial\lambda<0$. Counter-examples for this statement are provided by distributions that have $\mu'(x)>1$ and $d\log\mu(x)/d\log x \leq 1$ for every $x\in<\alpha,\beta>$, because from Proposition 1 it follows that in such cases $\partial\theta/\partial\lambda\geq 0$ for every possible $\phi\in<\alpha,\beta>$ ($\partial\theta/\partial\lambda\geq 0$ holds trivially if $\phi<\alpha$ or $\phi>\beta$) and therefore for every possible $\rho>0$. Take for instance the Pareto family with parameters $\nu>1$ and $w_0>0$. Then for every $x>w_0$ $\mu'(x)=\nu/(\nu-1)>1$ and $d\log\mu(x)/d\log x = 1$ (in fact, $\partial\theta/\partial\lambda$ is in this case given by $w_0.\rho.b.\phi^{-\nu-1}/(\rho+\theta)$ if $\phi>w_0$, which is clearly non-negative for every $\rho>0$).

5.3.2. *Defectiveness and the failure rate*

This subsection deals with the relationship between whether a distribution is defective or not on the one hand, and the shape of its failure rate for large values on the other. We examine distributions F satisfying Assumption 1, except that now we take $\beta=\infty$ and allow for defective F. Because of this we also skip the requirement that $E_W(w)<\infty$. The distribution is non-defective if

$\lim_{w\to\infty} F(w) = 1$; this is equivalent to

$$(6) \qquad \text{there is an } x\in<0,\infty> \text{ such that } \int_x^\infty \psi(w)dw = \infty$$

For this it is sufficient that there exist c and x ($0<c,x<\infty$) such that for every $w>x$ $\psi(w)\geq c.w^{-1}$. If, on the other hand, there exist ε,c and x ($0<\varepsilon,c,x<\infty$) such that for every $w>x$ $\psi(w)\leq c.w^{-1-\varepsilon}$ then the integral in (6) converges and F is defective. There is a strong relationship between these sufficient conditions for F to be (non–)defective and the value of the elasticity of $\psi(w)$ with respect to w for large w. Because of this we can state directly a relationship between this elasticity and the (non–)defectiveness of F.

Proposition 2.

Let F *be a distribution satisfying the assumptions stated at the beginning of this subsection. If there exists an* $x\in<0,\infty>$ *such that for every* $w>x$ $d\log\psi(w)/d\log w \geq -1$ *then* F *is non–defective. If there exist* ε *and* x ($0<\varepsilon,x<\infty$) *such that for every* $w>x$ $d\log\psi(w)/d\log w < -(1+\varepsilon)$ *then* F *is defective.*

The proof is in the appendix. Proposition 2 implies a link between Condition IIa and the (non–)defectiveness of the wage offer distribution F (if $\beta=\infty$). Specifically, if Condition IIa is satisfied, then F is non–defective. If Condition IIa is not satisfied in the sense that there exist ε and x ($0<\varepsilon,x<\infty$) such that for every $w>x$ $d\log\psi(w)/d\log w < -(1+\varepsilon)$ then F is defective. By restricting attention to non–defective F such violations of Condition IIa are ruled out. Therefore, if Condition IIa does not hold in the sense that there exists an x such that for every $w>x$ $d\log\psi(w)/d\log w < -1$ (which implies that Condition Ia is not satisfied for all $\phi>x$, see the appendix) and F is non–defective and

$$\lim_{w\to\infty} \frac{d \log \psi(w)}{d \log w} \text{ exists}$$

then it follows that this limit has to equal –1 and that the limit is approached from below.

Note that the link between Condition IIa and the (non–)defectiveness of F (if $\beta=\infty$) is based on the behaviour of the function $\psi(x)$ as $x\to\infty$. If $\psi(x)$ satisfies the inequality in Condition IIa for all sufficiently large values of x but not for all small values of x, then F is non–defective but F does not satisfy Condition IIa. (Such distributions can be constructed by making dips or peaks in the left part of the density of distributions satisfying Condition IIa.) On the other hand, if attention is restricted to $\psi(x)$ that are sufficiently smooth on $<\alpha,\infty>$ then the link between Condition IIa and the (non-)defectiveness of F can be strengthened. For instance, for the class of distributions F for which $\psi(x)=c.x^{\gamma}$ on $<\alpha,\infty>$ for some $\alpha,c>0$ and $\gamma\in\mathbb{R}$, it holds that Condition IIa is equivalent to the condition that F is non-defective.

5.3.3. *Violations of the conditions*

The previous subsections showed that the class of wage offer distributions for which in every case $\partial\theta/\partial\lambda\geq0$ is very large. Still, it might be interesting to examine which distributions violate the conditions for $\partial\theta/\partial\lambda$ to be non-negative and in which cases $\partial\theta/\partial\lambda$ is likely to be negative if the conditions are violated.

To start with the latter, if Condition Ia is not satisfied for certain $\phi\in<\alpha,\beta>$ and F, then one can choose b=0 and $\rho>0$ such that ϕ follows from equation (1) and $\partial\theta/\partial\lambda<0$. To obtain more results we assume that ψ is ultimately monotone; that is, there is an $N<\beta$ such that there is no sign-reversal of ψ' on $<N,\beta>$. Now suppose that the wage offer distribution F does not satisfy Condition Ia for any $\phi\in<\alpha,\beta>$. This is only possible if $\beta=\infty$, for if $\beta<\infty$ then Condition Ia holds for ϕ sufficiently close to β, which implies that $\partial\theta/\partial\lambda\geq0$ if ϕ is sufficiently close to β. Also, if $\alpha=0$ then Condition Ia holds for most ϕ sufficiently close to 0. (These statements are proved in the appendix.) So, suppose F has $\alpha>0$ and $\beta=\infty$ and F does not satisfy Condition Ia for any $\phi\in<\alpha,\beta>$. If ϕ is small due to a b that is almost zero (though $\phi>\alpha$) then $\partial\theta/\partial\lambda<0$. On the other hand, if ϕ is sufficiently large due to a sufficiently large b, then $\partial\theta/\partial\lambda\geq0$ (this is shown in the appendix). Thus, it seems that if F is such that $\partial\theta/\partial\lambda<0$ is possible, then the latter only occurs for small ϕ. Note however that things can be made more complicated by allowing explanatory variables other than b to vary. For instance, if λ is almost zero, then in most cases $\partial\theta/\partial\lambda\geq0$. In any case, the results in this paragraph shade the remark in Vroman (1985) that $\partial\theta/\partial\lambda<0$ is likely to occur only for large ϕ if at all.

The link between the failure rates of the distribution of wage offers w and the distribution of log wage offers z that was established in Subsection 5.3.1 provides a means of constructing wage offer distributions not satisfying Condition IIa. By choosing a distribution F_z for z that does not satisfy Condition 2a one obtains a distribution F_w for w that fails to satisfy Condition IIa. Such a distribution F_z has to satisfy certain restrictions in order to be able to generate an F_W satisfying Assumption 1. In particular, $E_Z(e^Z)$ has to be finite to ensure that $E_W(w)$ is finite. This restriction implies that most of the distributions that were shown in Section 5.2 to have a decreasing $\psi(x)$ for some x cannot be used as a model of F_Z in order to generate an F_W not satisfying Condition IIa. The exceptions are the members of the gamma family for which $-1<\gamma<0$ and $\lambda>1$ and the members of the log-uniform family. If z has a gamma distribution with $-1<\gamma<0$ and $\lambda>1$ then w has a log-gamma distribution with density

$$f_W(x) \sim (\log x)^{\gamma}.x^{-\lambda-1} \qquad x\in<1,\infty>$$

while if z has a log-uniform distribution with parameters α and β then w has a 'log-log-uniform' distribution with density

$$f_W(x) \sim (\log x)^{-1}.x^{-1} \qquad x\in<e^{\alpha},e^{\beta}>$$

These distributions satisfy Assumption 1. If w has a log-gamma distribution then $d\log\psi_W(x)/d\log x < -1$ for all $x\in<1,\infty>$ while if w has a 'log-log-uniform' distribution then this inequality holds for sufficiently small x if $1 + \log \alpha < \log \beta$. Other examples of violations of Condition IIa can be obtained by examining F_Z for which $\psi_Z(x)$ can be written as $c+g(x)$ with g continuously differentiable and $g(x)>0$ and $g'(x)<0$ for every $x\in<\alpha,\infty>$, in which $\alpha>-\infty$ is the lower bound of the interval of support of F_Z. If $c>1$ then $E_Z(e^Z)$ is finite. The corresponding F_W then satisfies Assumption 1 while $d\log\psi_W(x)/d\log x < -1$ for all x in the interval of support of F_W which is $<e^{\alpha},\infty>$. It can be shown that for these distributions F_W as well as for the log-gamma distributions with $-1<\gamma<0$ and $\lambda>1$ there holds that the limit of $d\log\psi_W(x)/d\log x$ as $x\to\infty$ equals -1.

We now show that if one translates a distribution satisfying Condition Ia but not Condition 1a sufficiently far to the right, then one obtains a distribution that does not satisfy the conditions listed in Proposition 1. Let a subscript y ($y\geq0$) denote the number of units that the original distribution is translated to the right. It follows that $\alpha_y=\alpha_0+y$, $\beta_y=\beta_0+y$, $f_y(x)=f_0(x-y)$, $\psi_y(x)=\psi_0(x-y)$ and $\mu_y(x)=y+\mu_0(x-y)$. Consequently,

$$(7) \qquad \left.\frac{d\ \log(\mu_y(x)-x)}{d\ \log x}\right|_{x=\phi} = \frac{\phi}{\phi-y}\cdot\left.\frac{d\ \log(\mu_0(x)-x)}{d\ \log x}\right|_{x=\phi-y} \qquad \phi\in<\alpha_y,\beta_y>$$

Note that if F_0 satisfies Assumption 1 then so does F_y. Suppose F_0 does not satisfy Condition 1a but does satisfy Condition Ia, for every $\phi \in \langle \alpha_0, \beta_0 \rangle$. Then for every $\phi \in \langle \alpha_y, \beta_y \rangle$

$$0 < \left. \frac{d \log(\mu_0(x)-x)}{d \log x} \right|_{x=\phi-y} \leq 1$$

On the other hand, the first part of the r.h.s. of equation (7) exceeds 1 and increases in y. Consequently, if ϕ–y is held constant then for sufficiently large ϕ and y the l.h.s. of (7) exceeds 1 and Condition Ia is not satisfied. Now recall from Subsection 5.3.1 that Condition Ia is not invariant under simultaneous and equally large shifts of F and b, though $\partial\theta/\partial\lambda$ is. Therefore, if we translate F_0 to the right by y units and hold ϕ–y constant by shifting b correspondingly (that is, by increasing b with y units), and if y is sufficiently large, then Condition Ia is not satisfied but $\partial\theta/\partial\lambda$ remains non–negative. On the other hand, as shown at the beginning of this subsection, if Condition Ia is not satisfied for certain ϕ and F then there always values of b, λ and ρ such that ϕ is the reservation wage corresponding to these values and $\partial\theta/\partial\lambda<0$.

As an example, assume that F_0 is a Pareto distribution with parameters $w_0>0$ and $\nu>1$. Then, for every $y>0$ and $\phi>w_0+y$, $d\log(\mu_y(\phi)-\phi)/d\log\phi = \phi/(\phi-y)>1$, so Condition Ia is not satisfied for any $\phi>w_0+y$ if $y>0$. By relating this to the discussion in Subsection 5.3.1 on a necessary and sufficient condition for $\partial\theta/\partial\lambda\geq 0$ it follows that if $b<y$ and $\phi>w_0+y$ then $\partial\theta/\partial\lambda<0$, while if $b=y$ and $\phi>w_0+y$ then $\partial\theta/\partial\lambda=0$. In fact, for $y\geq 0$,

$$\frac{\partial\theta}{\partial\lambda} = \frac{\rho}{\rho+\theta} \cdot \frac{w_0^{\nu}}{(\phi-y)^{\nu+1}} \cdot (b-y) \qquad \text{if } w_0+y < \phi < \infty$$

Consequently, if $\phi>w_0+y$, then $\partial\theta/\partial\lambda<0$ if and only if $b<y$. One can show that if $y>0$ and $\lambda<\rho.(\nu-1)$, then $b<y$ implies that $\phi<w_0+y$, so in those cases always $\partial\theta/\partial\lambda\geq 0$ whatever the value of y. For the translated Pareto distributions, the limit of $d\log\psi_w(x)/d\log x$ as $x\to\infty$ obviously equals –1 since translation has no effect in the limit.

Other violations of the conditions listed in Proposition 1 can be found by examining non–smooth distributions. From the interpretation of Condition Ia in terms of the magnitude of the wage offer density f at ϕ, it follows that if one makes a sufficiently high peak in the density of a distribution, and if $f(\phi)$ is at that peak, then the resulting distribution does not satisfy Condition Ia, and $\partial\theta/\partial\lambda<0$. Also, recall the remarks at the end of Subsection 5.3.2 about comparable violations of Condition IIa. Note that this suggests that for every distribution satisfying Conditions Ia or IIa there are distributions that deviate from it only on a small interval, but do not satisfy these conditions.

From the results in this subsection it follows that Condition Ia is translation dependent in the sense that there are F that satisfy Condition Ia for all possible $\phi \in \langle \alpha,\beta \rangle$, while translations of such F do not satisfy Condition Ia for certain ϕ in the domain of the translated F. From the Pareto example it also follows that Conditions IIa and IIIa are translation dependent in the sense that there are F satisfying them for which translation results in a distribution not satisfying them and vice versa. This implies that the conditions listed in Proposition 1 cannot be characterized in terms of moment restrictions. Also note that, since all Pareto distributions with parameter $\nu>1$ satisfy the conditions, one can always find distributions for which moments of order larger than $1+\varepsilon$ $(\varepsilon>0)$ do not exist, that satisfy the conditions.

5.4. The effect of an increase of the benefits level on the hazard

In this section we examine the implications of Condition IIa for the relationship between θ and b, which of course is a negative relationship if $\alpha<\phi<\beta$. It follows from equations (1) and (2) that if $\alpha<\phi<\beta$ then

$$(8) \qquad \left|\frac{\partial \log \theta}{\partial \log \mathrm{b}}\right| = \mathrm{b}.\psi(\phi).\frac{\rho}{\rho+\theta}$$

Clearly, if F satisfies Condition 2a, then this expression does not decrease as b increases. By rewriting (8) it becomes obvious that this result can be generalized to F satisfying Condition IIa.

$$(9) \qquad \left|\frac{\partial \log \theta}{\partial \log \mathrm{b}}\right| = [\phi.\psi(\phi)] \cdot \frac{\mathrm{b}}{\phi} \cdot \frac{\rho}{\rho+\theta}$$

If Condition IIa is satisfied then the first term of the r.h.s. of (9) does not decrease as b (and therefore ϕ) increases. (Note that if $f'(\phi)>0$ then $\phi.\psi(\phi)$ increases in b and ϕ, regardless of whether Condition IIa holds or not.) The second term of the r.h.s increases as b increases, which can be shown by differentiation, using $\partial\phi/\partial \mathrm{b}=\rho/(\rho+\theta)$. Because the third term also increases as b increases, the result follows. From this it is also obvious that $|\partial \log\theta/\partial \log \mathrm{b}|$ is not decreasing in b under conditions more general than Condition IIa. Moreover, in Section 5.5 it is shown that this result holds in models more general than considered here. Consequently, under fairly general conditions the absolute value of the elasticity of the hazard with respect to the level of benefits is non–decreasing in b.

In reduced–form models of unemployment duration, log θ is generally specified to be linear in log b, so $\partial \log\theta / \partial \log b$ is a constant. From the previous paragraph it follows that according to job search theory the estimate of the elasticity of θ with respect to b is likely to be biased towards zero for individuals with a high level of benefits and biased away from zero for individuals with a low level of benefits. This bias may be reduced substantially by adding $(\log b)^2$ as a regressor in log θ.

5.5. Extensions of the model

5.5.1. *Utility maximization*

In this subsection it is shown that the results derived in Section 5.3 are also valid in more general models of job search. Specifically, in models in which the assumption of income maximization is weakened by allowing individuals to maximize utility, the conditions listed in Proposition 1 remain sufficient for $\partial\theta/\partial\lambda \geq 0$ for a wide range of utility flow functions.

Suppose unemployed individuals maximize expected discounted lifetime utility (over an infinite horizon). We assume that utility is intertemporally separable, the utility flow function u being a function of the income flow. (One can allow for utility flow functions that also depend separably on the present labour market state without affecting the results.) The following assumptions on λ, F(w), b, ρ and u replace Assumptions 1 and 2.

1–u. *equals Assumption* 1 *except that here* $E_W(u(w))<\infty$ *instead of* $E_W(w)<\infty$.

2–u. *equals Assumption* 2.

3–u. $u(x)$ *is twice differentiable on* $<0,\infty>$. *For every* $x\in<0,\infty>$ $u'(x)>0$. *Further*, $u(b)\geq 0$ *and* $u(\beta)>0$.

The reservation wage ϕ is the unique solution of

$$(10)\qquad u(\phi) = u(b) + \frac{\lambda}{\rho}.\int_{\phi}^{\infty}(u(w)-u(\phi))dF(w)$$

From the assumptions above it follows that $0<u(\phi)<\infty$ and $0<\phi<\infty$. Again we restrict attention to cases in which $\alpha<\phi<\beta$. Let $\eta(x)$ denote the expected value of u(w) conditional on w>x as a function of x, so $\eta(x)=E_w(u(w)|w>x)$. The expression $\eta(\phi)$ equals the expected utility flow in employment for an individual with reservation wage ϕ. If u is linear then of course $\eta(x)=\mu(x)$.

Proposition 3.

In a job search model that satisfies Assumptions 1–u, 2–u *and* 3–u *and in which* $\alpha<\phi<\beta$, *sufficient conditions for* $\partial\theta/\partial\lambda$ *to be non–negative are*

Ia–u $\quad \dfrac{d \log \eta(x)}{d \log x} \leq \dfrac{d \log u(x)}{d \log x}$ *at* $x=\phi$.

IIa–u $\quad \psi(x).u(x)/u'(x)$ *does not decrease on* $<\alpha,\beta>$.

There holds that IIa–u $\Rightarrow$ Ia–u.

The proof is completely analogous to the proof of Proposition 1 and is therefore omitted. Although conditions similar to Conditions Ib, IIb and IIIa can be derived, they are not presented here since they are not very informative or elegant. Condition IIa–u does not make any reference to the actual value of ϕ; hence, if this condition holds then $\partial\theta/\partial\lambda\geq0$ for all admissible values of b, λ and ρ.

One may say somewhat loosely that Condition Ia–u states that if the reservation wage ϕ slightly increases, then the proportional increase of the expected utility flow in employment does not exceed the proportional increase of the utility flow associated with an income equal to ϕ. For instance, an increase of the reservation wage such that $u(\phi)$ increases by 10% implies an increase of the expected utility flow in employment of at most 10%.

If $u(\alpha)\geq0$, then Condition IIa–u, as a condition on F, is weaker than Condition IIa if and only if $u(x)/(x.u'(x))$ does not decrease on $<\alpha,\beta>$. The latter is equivalent to the requirement that $\log u(e^x)$ is a concave function of x on $<\log\alpha,\log\beta>$. Further, if $u(\alpha)\geq0$ then Condition IIa–u, as a condition on F, is weaker than Condition 2a if and only if $u(x)/u'(x)$ does not decrease on $<\alpha,\beta>$. The latter holds if and only if $\log u(x)$ is a concave function of x on $<\alpha,\beta>$. Sufficient for this is that u(x) is concave on $<\alpha,\beta>$ or that $\log u(e^x)$ is concave on $<\log\alpha,\log\beta>$.

In the appendix we examine under what circumstances Condition IIa–u is satisfied for some well–known and popular classes of utility flow functions. It appears that for all utility flow functions considered, the conditions on the shape of F for $\partial\theta/\partial\lambda$ to be non–negative in search models with utility maximization are (even) weaker than the corresponding conditions in models with income maximization.

It may be interesting to examine under what conditions the results in Section 5.4 carry over to models with utility maximization. It follows from equations (10) and (2) that if $\alpha<\phi<\beta$ then

$$(11) \qquad \left|\frac{\partial \log \theta}{\partial \log b}\right| = [\phi.\psi(\phi)] \cdot \frac{b.u'(b)}{\phi.u'(\phi)} \cdot \frac{\rho}{\rho+\theta}$$

Again, the third term of the r.h.s. of (11) increases as b increases while if Condition IIa is satisfied then the first term of the r.h.s. does not decrease as b increases. It seems to be impossible to give a simple condition in terms of the utility flow function that ensures that the second term does not decrease as b increases. However, in the appendix we check whether this term does not decrease in b for some popular classes of utility flow functions. The results reinforce the conclusions of Section 5.4.

5.5.2. *Nonstationarity*

In Section 5.3 and Subsection 5.5.1 we have examined the derivative of the job offer arrival rate λ with respect to the hazard θ in stationary job search models. The results can be used for interpreting the implications for θ of individual differences with respect to λ. Alternatively, the results represent the comparative statics of θ with respect to λ, i.e. they show how the stationary (or, equilibrium) values of θ differ when the arrival rates λ differ. It may also be interesting to examine the time path of θ when λ is changing for an individual. It is widely believed that λ is duration dependent because of a so-called scar effect: employers may think that long-term unemployed individuals are less productive than short-term unemployed individuals (see e.g. Narendranathan, Nickell & Stern (1985)). Often, negative duration dependence of θ in an estimated reduced-form model of unemployment duration is attributed to this scar effect (see e.g. Jensen & Vishwanath (1985)). Therefore it seems to be particularly interesting to examine under what conditions θ is a decreasing function of unemployment duration if λ is a decreasing function of duration.

If λ, b or F(w) change during unemployment then the model is nonstationary. We consider nonstationarity that arises because λ is a decreasing function of duration t. (For ease of exposition we let calendar time and unemployment duration coincide, so that time dependence of λ and duration dependence of λ can be considered simultaneously.) Further, we will be concerned with job searchers who correctly anticipate changes in the value of λ. The following assumptions on λ, F(w), b and ρ replace Assumptions 1 and 2.

1–n. *equals Assumption* 1.

2–n. $0<\rho<\infty$, $0\leq b<\infty$.

3–n. *For every* $t\in[0,\infty>$, $0<\lambda(t)<K<\infty$, K *being a fixed number. There exists some number* $T\in<0,\infty>$ *such that* $\lambda(t)$ *is constant on* $[T,\infty>$ *and* $\lambda(t)$ *is differentiable on* $[0,T>$. *For every* $t\in[0,T>$, $\lambda'(t)\leq 0$. *The left-hand limits* $\lambda_L(T)$ *and* $\lim_{t\uparrow T} \lambda'(t)$ *of* $\lambda(t)$ *and* $\lambda'(t)$ *at* T *exist. There holds that* $\lambda_L(T)\geq\lambda(T)$.

From Chapter 3 it follows that if Assumptions 1–n, 2–n and 3–n are satisfied, then the optimal strategy of a job searcher can be characterized by a unique, bounded and continuous reservation wage function $\phi(t)$ on $[0,\infty>$ which satisfies the following differential equation for every $t\in[0,T>\cup<T,\infty>$.

(12) $\phi'(t) = \rho.\phi(t) - \rho.b - \lambda(t).Q(\phi(t))$

For $t\geq T$ the model is stationary and $\phi(t)$ follows by imputing $\phi'(t)=0$ in equation (12). If λ is continuous at T (so $\lambda_L(T)=\lambda(T)$) then $\phi'(T)=0$, otherwise the right-hand derivative $\phi_R'(T)$ of ϕ with respect to t at T equals zero while the left-hand derivative $\phi_L'(T)$ can be calculated by replacing $\lambda(T)$ in the r.h.s. of equation (12) at T by $\lambda_L(T)$. It can be shown that for every $t\in[0,\infty>$ $\phi(t)>b$. For the same reason as before, attention is restricted to cases in which for every $t\in[0,\infty>$ $\alpha<\phi(t)<\beta$.

From Chapter 3 it also follows that for every $t\in[0,T>$ $\phi'(t)\leq 0$. (This can be strengthened to $\phi'(t)<0$ if for every $t\in[0,T>$ $\lambda'(t)<0$. However, for ease of exposition we will only present results in terms of weak inequalities.) If $\lambda_L(T)>\lambda(T)$ then $\phi_L'(T)<0$.

If $\lambda(t)$ is constant on $[0,T>$ and $\lambda(0)>\lambda(T)$ (so the only source of nonstationarity is a discrete downward jump of λ at T) then additional results can be derived. Note that in such cases $\phi(t)$ satisfies a homogeneous constant-coefficient differential equation on $[0,T>$. Let ϕ_1 denote the stationary solution of the differential equation (12) on $[0,T>$. One can interpret ϕ_1 as the optimal reservation wage on $[0,T>$ if λ would not jump at T. It can be shown (see Chapter 3) that for every $t\in[0,T>$ $\phi(T)<\phi(t)<\phi_1$, $\phi'(t)<0$ and $\phi''(t)<0$ and that $\phi_L'(T)<0$.

The hazard θ is a function of unemployment duration t,

(13) $\theta(t) = \lambda(t).\bar{F}(\phi(t))$

Because for every $t \geq T$ $\lambda(t)=\lambda(T)>0$ and $\phi(t)=\phi(T)<\beta$, it follows that for every $t \geq T$ $\theta(t)=\theta(T)>0$ and therefore the duration distribution is non-defective. In fact, for every $t \geq 0$, $\theta(t)>0$. The Assumptions 1-n, 2-n and 3-n and the results on ϕ imply that $\theta(t)$ is differentiable on [0,T> and constant on [T,∞>. If $\lambda(t)$ is discontinuous at T ($\lambda_L(T)>\lambda(T)$) then so is $\theta(t)$ ($\theta_L(T)>\theta(T)$, note that $\theta_L(T)$ exists) and vice versa. Also, if $\lambda(t)$ is differentiable at T then so is $\theta(t)$ and vice versa. Finally, therefore, if $\lambda(t)$ is continuous but not differentiable at T then so is $\theta(t)$. Note that if $\theta(t)$ is not differentiable then still $\lim_{t \uparrow T} \theta'(t)$ exists.

Using a result from Chapter 3, one can show that if Assumptions 1-n, 2-n and 3-n are satisfied and if Condition IIa holds, then for every $t \leq T$ $\theta(t) \geq \theta(T)$. Let, for every $t \in$[0,T>, $\phi_0(t)$ denote the reservation wage at t that is optimal if λ is constant after t, i.e. if for every $\tau \geq t$ $\lambda(\tau)=\lambda(t)$. From Chapter 3 it follows that $\phi_0(t) \geq \phi(t)$. This implies that the hazard $\theta_0(t)$ corresponding to the case in which for every $\tau \geq t$ $\lambda(\tau)=\lambda(t)$ holds, satisfies $\theta_0(t) \leq \theta(t)$. Now note that both $\theta_0(t)$ and $\theta(T)$ can be interpreted as hazards in stationary models, the only difference between the models being the arrival rate, which is $\lambda(t)$ in the first case and $\lambda(T)$ in the second. Because $\lambda(t) \geq \lambda(T)$, Condition IIa is sufficient for $\theta_0(t) \geq \theta(T)$. Consequently, if Condition IIa holds then $\theta(T) \leq \theta_0(t) \leq \theta(t)$, and the result follows.

The main objective of this subsection is to examine under what conditions on $\lambda(t)$ it holds that $\theta(t)$ is a decreasing function of t on [0,T]. By differentiation of equation (13) it follows that for every $t \in$[0,T>,

(14) $\theta'(t) = \lambda'(t).\bar{F}(\phi(t)) - \lambda(t).f(\phi(t)).\phi'(t)$

From this equation it is obvious that nonstationarity due to a decreasing λ has two opposite effects on the derivative of the hazard with respect to unemployment duration at t<T. First, there is a negative effect because of the instantaneous decrease of the arrival rate of job offers at t. Secondly, there is a positive effect because of the anticipation of future decreases of the arrival rate of job offers. This effect works in the following way. If λ decreases on <t,T> then the expected number of opportunities to accept a job offer in any time period with fixed length directly after t decreases as t increases. Therefore the expected discounted lifetime income decreases as time proceeds. This implies that individuals become less selective with regard to job offers as time proceeds, so ϕ decreases at t and this has a positive effect on $\theta'(t)$. Note that the instantaneous decrease of λ at t (the first effect) does not affect $\phi(t)$ (the second effect) at all because the probability to obtain a job offer at t is zero. This makes the analysis of $\theta'(t)$ fundamentally different from the analysis of $\partial\theta/\partial\lambda$ in a stationary model. For given $\phi(t)$, F and $\lambda(t)$ one can always choose $\lambda'(t)$ to make $\theta'(t)<0$ or to make $\theta'(t)>0$, without affecting $\phi(t)$ or $\lambda(t)$. Consequently, conditions ensuring that $\theta'(t)\leq 0$ always have to restrict $\lambda'(t)$ in some way. Or, in other words, conditions on F ensuring that $\theta'(t)\leq 0$ must be conditional on some condition that restricts the time path of $\lambda(t)$. The following proposition presents such conditions.

Proposition 4.

In a job search model that satisfies Assumptions 1–n, 2–n *and* 3–n *and in which for every* $t\geq 0$ $\alpha<\phi(t)<\beta$, *sufficient for* $\theta'(t)\leq 0$ *on* $[0,T>$ *is that for every* $t\in[0,T>$ *and for every* $x\in<\alpha,\beta>$ *there holds that*

$$(15) \qquad \mu'(x) \leq -\frac{\lambda'(t)}{\lambda^2(t)}$$

Further, $\lim_{t\uparrow T} \theta'(t) \leq 0$ *and* $\theta_L(T)\geq\theta(T)$. *Sufficient for equation* (15) *to hold for* $x\in<\alpha,\beta>$ *and* $t\in[0,T>$ *is that*

(16) *Condition* 2a *is satisfied and* $\lambda'(t) \leq -\lambda^2(t)$.

or that

(17) *Condition* IIa *is satisfied and* $\dfrac{\mu(x)}{x} \leq -\dfrac{\lambda'(t)}{\lambda^2(t)}$.

The proof is in the appendix. Jensen & Vishwanath (1985) proved that a condition analogous to (16) is sufficient for θ to be decreasing in a discrete-time model with positive search costs and zero benefits. The functions $\lambda(t)$ satisfying (16) with an equality sign is $\lambda(t)=1/(t+k)$ with $k>0$.

Clearly, there is a trade-off between the condition on F and the condition on $\lambda(t)$. If $\lambda(t)$ decreases sharply then, for most F, equation (15) is satisfied whereas if $\lambda(t)$ decreases moderately then the class of F for which (15) is satisfied is restrictive.

Let us return to the cases in which $\lambda(t)$ is constant on $[0,T>$ and $\lambda(0)>\lambda(T)$. In such cases $\phi'(t)<0$ on $[0,T>$, so, from equation (14), for every

$t \in [0,T>$ $\theta'(t)>0$, $\lim_{t \uparrow T} \theta'(t) > 0$ and $\theta_L(T)>\theta(T)$. Consequently, $\theta(t)$ increases as t goes from 0 to T but jumps downward at T. From the results above it follows that Condition IIa is sufficient for $\theta(T) \leq \theta(0)$, so it is well possible that in the long run θ is smaller than than the θ during the first days of unemployment. However, it is important to note that $\theta(t)$ does not move from $\theta(0)$ to $\theta(T)$ in a monotone way. In the general case, if $\lambda(t)$ is more or less constant while $\phi(t)$ decreases because of strong future decreases of λ, then $\theta(t)$ increases. It is well conceivable that during the first weeks of unemployment the so-called scar effects play no part and λ is constant while after that λ decreases. If the latter scenario is true then a monotonically decreasing hazard rate cannot be explained solely by duration dependence of λ. In sum, it appears both from this paragraph and from Proposition 4 that the conditions needed for a monotonically decreasing hazard that is due to a monotonically decreasing arrival rate are too strong to be acceptable a priori.

In this chapter we have examined under what conditions θ is an increasing function of λ in stationary models, and under what conditions θ is a decreasing function of duration t when λ is a decreasing function of t, in nonstationary models. A topic for further research would be to examine the comparative dynamics of θ with respect to λ in nonstationary models; that is, to examine how the function $\theta(t)$ changes when the whole function $\lambda(t)$ is shifted upward.

5.6. Conclusion

In this chapter we have examined the sign of the effect of an increase of the job offer arrival rate on the hazard of the unemployment duration distribution. It was shown that previously derived sufficient conditions on the wage offer distribution for this effect to be positive in job search models can be weakened considerably at no cost, to include virtually every conceivable wage offer distribution. In particular, all families of distributions generally used to model wage offer distributions in structural job search models and other income-related distributions do not satisfy the conditions derived before but do satisfy the conditions presented in this chapter.

The results have some implications for both structural and reduced-form empirical analyses of unemployment duration. First of all, the interpretation of the estimates of reduced-form models is facilitated. If an explanatory variable is found to have a positive influence on the hazard and this variable is believed not to influence determinants of the hazard other than the job offer arrival rate (e.g. number of working individuals in the household), then it can be concluded that the variable has a positive influence on the job offer arrival rate. Also, if it is believed that the influence of an explanatory variable on the hazard mainly acts by way of the arrival rate (e.g. local unemployment percentage) then reduced-form estimates can be used to check whether the data are in agreement with these prior beliefs. Another implication concerns the estimation of structural job search models. The class of distributions satisfying the sufficient conditions presented in this chapter is very large, so it is likely that small departures from the families assumed in applications do not result in distributions not satisfying the sufficient conditions. Therefore, it is likely that slight changes of the shape of the wage offer distribution do not imply a sign-reversal of the relationship between the arrival rate and the hazard. Consequently, one may say that in this sense the estimates of structural models are insensitive with respect to the assumed family of wage offer distributions.

The analysis in this chapter generated some by-products which may be of independent interest. It is shown that generally in job search models the absolute value of the elasticity of the hazard with respect to the level of benefits is an increasing function of the level of benefits. This suggests that in reduced-form empirical analyses the log hazard should be allowed to be a non-linear function of log benefits. It is also shown that, in nonstationary job search models in which the arrival rate is a decreasing function of duration, the conditions on the wage offer distribution and the time path of the arrival rate that are needed to obtain a hazard that is a monotonically decreasing function of duration are too strong to be acceptable a priori.

Appendix to Chapter 5

5.A.1. Proof of Proposition 1

We start by restating the equation linking Q,$\overline{\mathrm{F}}$ and μ,

$$\text{(A1)} \qquad Q(x) = \overline{F}(x).(\mu(x)-x)$$

Because $\alpha<\phi<\beta$, equations (1) and (2) imply that

$$\text{(A2)} \qquad \frac{\partial\theta}{\partial\lambda} = \overline{F}(\phi) - \lambda.f(\phi).\frac{Q(\phi)}{\rho+\theta} = \frac{1}{\rho+\theta}\,[(\rho+\theta)\overline{F}(\phi) - \theta.f(\phi).(\mu(\phi)-\phi)]$$

The latter equality follows because of equation (A1). Consequently, $\partial\theta/\partial\lambda\geq 0$ if and only if

$$\text{(A3)} \qquad \frac{\rho}{\theta} \geq \frac{f(\phi)}{\overline{F}(\phi)}.(\mu(\phi)-\phi) - 1$$

(note that θ and $\overline{F}(\phi)$ are positive). Equation (1) can be rewritten as

$$\phi-b = \frac{\theta}{\rho}.(\mu(\phi)-\phi)$$

so inequality (A3) can be rewritten as

$$\frac{\mu(\phi)-\phi}{\phi - b} \geq \frac{f(\phi)}{\overline{F}(\phi)}\,.(\mu(\phi)-\phi) - 1$$

Because $b\geq 0$, sufficient for this is that

$$\frac{\mu(\phi)-\phi}{\phi} \geq \frac{f(\phi)}{\overline{F}(\phi)}.(\mu(\phi)-\phi) - 1$$

or, equivalently,

$$\text{(A4)} \qquad \frac{\mu(\phi)}{\phi} \geq \psi(\phi).(\mu(\phi)-\phi)$$

For every $x\in<\alpha,\beta>$ there holds that

$$\mu'(x) = \frac{d}{dx}\left[\frac{\int_x^\beta wf(w)dw}{\overline{F}(x)}\right] = \psi(x).(\mu(x)-x)$$

So inequality (A4) is equivalent to $\mu(\phi) \geq \phi.\mu'(\phi)$. Of course the latter is just another way of writing Condition Ia. Because $\mu(\phi)$ exceeds ϕ, Condition Ia is weaker than Condition 1a (which states that $\mu'(\phi)\leq 1$). Note that inequalities (A3) and (A4) are equivalent if and only if b=0, so only in that case Condition Ia is necessary and sufficient for $\partial\theta/\partial\lambda$ to be non-negative.

Condition Ib is satisfied if and only if the derivative of log $Q(e^X)$ with respect to x is non-increasing at x = log ϕ. This derivative equals $-e^X.\overline{F}(e^X)/Q(e^X)$. Since e^X is a strictly increasing function, Condition Ib is satisfied if and only if $x.\overline{F}(x)/Q(x)$ is non-decreasing at $x=\phi$. Because of equation (A1) the latter holds if and only if $x/(\mu(x)-x)$ is non-decreasing at $x=\phi$, which can easily shown to be equivalent to Condition Ia.

By multiplying both sides of inequality (A4) with $\phi.\overline{F}(\phi)$ and by using equation (A1) one obtains

$$\int_{\phi}^{\beta} wf(w)dw \geq \phi.\psi(\phi).Q(\phi)$$

which can be rewritten as

$$\int_{\phi}^{\beta} w\psi(w)\overline{F}(w)dw \geq \phi.\psi(\phi).\int_{\phi}^{\beta} \overline{F}(w)dw$$

So Condition Ia is equivalent to

$$\text{(A5)} \qquad \int_{\phi}^{\beta}(w\psi(w) - \phi\psi(\phi)).\overline{F}(w)dw \geq 0$$

Sufficient for (A5) to hold is that $w.\psi(w)$ increases on $<\alpha,\beta>$. The latter is equivalent to Condition IIa. Clearly, the requirement that $w.\psi(w)$ increases is weaker than the requirement that $\psi(w)$ increases, so Condition IIa is weaker than Condition 2a.

Equation (A4) is derived by using the assumption that $b\geq 0$. More generally, if one assumes that $b\geq\pi$ for some π, then the left-hand side of (A4) can be replaced by $(\mu(\phi)-\pi)/(\phi-\pi)$ (note that $\phi>b\geq\pi$). One can show that sufficient for the revised equation (A4) to hold is that $(w-\pi).\psi(w)$ increases on $<\phi,\beta>$. The latter holds if and only if $d\log\psi(w)/d\log w \geq -w/(w-\pi)$ for every $w\in<\phi,\beta>$, which in turn is weaker than Condition IIa for every $\pi>0$. However, in the sequel we will stick to $\pi=0$ since this makes the weakest assumption on b.

Condition IIb is satisfied if and only if the derivative of $\overline{F}(e^X)$ with respect to x does not increase on $<-\infty,\log\beta>$. This derivative equals

$-e^x.f(e^x)/\overline{F}(e^x)$. Since e^x is strictly increasing, this is satisfied if and only if $xf(x)/\overline{F}(x)$ does not decrease on $<-\infty,\beta>$, which is just another way of writing Condition IIa.

From the definition of $\overline{F}$,

$$\overline{F}(e^x) = \int_x^\infty e^w.f(e^w)dw$$

Karlin (1968) shows that if an integrable function g(x) is log concave then the function defined by the integral of x to infinity of g is also log concave. Consequently, sufficient for $\overline{F}(e^x)$ to be log concave is that $e^x.f(e^x)$ is log concave. However, it is easily seen that $e^x.f(e^x)$ is log concave if and only if $f(e^x)$ is log concave. In sum, Condition IIIa implies Condition IIb. Note that this method of proof can also be used to prove that Condition IIb implies Condition Ib.

It remains to establish the relationship between Conditions IIIa and 3a. Condition IIIa is satisfied if and only if $e^x.f'(e^x)/f(e^x)$ does not increase on $<\log\alpha,\log\beta>$, which holds if and only if $xf'(x)/f(x)$ does not increase on $<\alpha,\beta>$. Likewise, Condition 3a is satisfied if and only if $f'(x)/f(x)$ does not increase on $<\alpha,\beta>$. Consequently, if for every $x\in<\alpha,\beta>$, $f'(x)\leq 0$ then Condition IIIa is weaker than Condition 3a. If for every $x\in<\alpha,\beta>$, $f'(x)\geq 0$ (which is possible only if $\beta<\infty$) then the reverse holds. In all other cases the conditions are not nested. Still, it always holds that Condition IIa is weaker than Condition 3a (because 3a $\Rightarrow$ 2a $\Rightarrow$ IIa).

5.A.2. Truncation from above

Examine a distribution satisfying Assumption 1 (with interval of support $<\alpha,\beta>$) that is truncated from above at $\overline{w}$ ($\alpha<\overline{w}<\beta$). For clarity, symbols referring to this truncated distribution are given a subscript *t*. There holds that $f_t(x)=f(x)/F(\overline{w})$ on $<\alpha,\overline{w}>$ and $\overline{F}_t(x)=(\overline{F}(x)-\overline{F}(\overline{w}))/F(\overline{w})$ on $<\alpha,\overline{w}>$. Therefore,

$$\psi_t(x) = \psi(x)\,.\frac{\overline{F}(x)}{\overline{F}(x)-\overline{F}(\overline{w})} \qquad\qquad x\in<\alpha,\overline{w}>$$

By differentiating this we obtain

$$\text{(A6)}\qquad \frac{d\log\psi_t(x)}{d\log x} = \frac{d\log\psi(x)}{d\log x} + x.\psi(x)\,.\frac{\overline{F}(\overline{w})}{\overline{F}(x)-\overline{F}(\overline{w})}$$

Because the second part of the right–hand side (r.h.s.) is positive it follows that the condition

$$\frac{d \log \psi_t(x)}{d \log x} \geq -1 \text{ for every } x \in \langle \alpha, \overline{w} \rangle$$

is weaker than Condition IIa. Further, the second part of the r.h.s. of (A6) increases as $\overline{w}$ decreases and it goes to ∞ if $\overline{w} \downarrow x$. So, if the first part of the r.h.s. of (A6) is <-1 (that is, the original distribution does not satisfy Condition IIa) then for sufficiently small $\overline{w}$ the left–hand side (l.h.s.) of (A6) is ≥ -1 for every $x \in \langle \alpha, \overline{w} \rangle$ (that is, the truncated distribution does satisfy Condition IIa). Alternatively, if ϕ is sufficiently large then the l.h.s. of (A6) is ≥ -1 for every $x \in [\phi, \overline{w} \rangle$. This is sufficient for Condition Ia to be satisfied for the truncated distribution (see Appendix 5.A.1) so in that case $\partial\theta/\partial\lambda \geq 0$.

5.A.3. Proof of Proposition 2

Suppose there is an x such that for every $w>x$ $d\log\psi(w)/d\log w \geq -1$. Then for every $w>x$ $d\log\psi(w)/dw \geq -1/w$, which implies that

$$\forall w>x, \quad \int_x^w \frac{d \log \psi(y)}{dy}\, dy \geq -\int_x^w \frac{1}{y}\, dy$$

This is equivalent to

$$\forall w>x, \quad \log \psi(w) - \log \psi(x) \geq \log x - \log w$$

which in turn is equivalent to

$$\forall w>x, \quad \psi(w) \geq x.\psi(x)/w$$

Consequently, there are x and c ($0<x,c<\infty$) such that for every $w>x$ $\psi(w) \geq c/w$. From the main text, this implies that F is non–defective. Suppose there are ε and x ($0<\varepsilon,x<\infty$) such that for every $w>x$ $d\log\psi(w)/d\log w \leq -(1+\varepsilon)$. Following the argument above we obtain

$$\forall w>x, \quad \psi(w) \leq x^{1+\varepsilon}.\psi(x)/w^{1+\varepsilon}$$

Again, from the main text, this implies that F is defective.

5.A.4. The sign of $\partial\theta/\partial\lambda$ if ϕ is large

F is non-defective if and only if

$$\int_{\alpha}^{\beta} \psi(x)dx = \infty$$

Because for every $x<\beta$, $\psi(x)<\infty$ this implies that if $\beta<\infty$ then it is necessary that $\lim_{x\uparrow\beta} \psi(x) = \infty$ for F to be non-defective. Consequently, if $\beta<\infty$ and $\psi(x)$ is ultimately monotone, then $\psi(x)$ increases at all points sufficiently close to β, which implies that the Conditions 1a and Ia hold for ϕ sufficiently close to β, so in such cases $\partial\theta/\partial\lambda\geq 0$.

Suppose that Condition Ia is not satisfied for any $\phi\in<\alpha,\beta>$, so $\mu'(\phi)>\mu(\phi)/\phi$ for any $\phi\in<\alpha,\beta>$, and that $\alpha=0$. There holds that $\mu(\phi)$ is positive and increasing on $<0,\beta>$. Let ε be a number between 0 and β. It follows that for every $\phi\in<0,\varepsilon>$, $\mu(\phi)>\mu(0)>0$ and therefore $\mu'(\phi)>\mu(0)/\phi$. By integrating the latter inequality from $\phi=x$ with $x\in<0,\varepsilon>$ to $\phi=\varepsilon$, one obtains that for every $x\in<0,\varepsilon>$ there holds that

$$\mu(x) < \mu(\varepsilon) + \mu(0).(\log x - \log \varepsilon)$$

If x is sufficiently small then the r.h.s. of this inequality is negative, so by contradiction it follows that for every $\varepsilon>0$ there are $\phi\in<0,\varepsilon>$ for which Condition Ia holds.

Now suppose $\beta=\infty$ and $\alpha>0$. If Condition Ia is not satisfied for any $\phi\in<\alpha,\beta>$ and $\psi(x)$ is ultimately monotone, then $\psi(x)$ has to be non-increasing at sufficiently large points. Consequently, ψ is uniformly bounded on $<\alpha,\infty>$. We use this result when examining the sign of $\partial\theta/\partial\lambda$ for large ϕ. From inequality (A3), $\partial\theta/\partial\lambda\geq 0$ if and only if

$$\frac{\rho}{\lambda} + \overline{F}(\phi) \geq \psi(\phi).[\overline{F}(\phi).(\mu(\phi)-\phi)]$$

Because of equation (A1), this is equivalent to

$$\text{(A7)} \qquad \frac{\rho}{\lambda} + \overline{F}(\phi) \geq \psi(\phi).Q(\phi)$$

If $b\to\infty$ then $\phi\to\infty$ while of course ρ, λ and the functions ψ, $\overline{F}$ and Q are unaffected. The lim $\phi\to\infty$ of the l.h.s. of (A7) equals ρ/λ and is therefore strictly positive. Because $Q(\phi)$ tends to zero as $\phi\to\infty$ and $\psi(\phi)$ is uniformly bounded, the lim $\phi \to \infty$ of the r.h.s of (A7) equals zero. Because $\bar{F}$, ψ and Q are continuous in their arguments, this implies that whatever the shape of the wage offer distribution, inequality (A7) is always satisfied for sufficiently large ϕ. Note that from equation (A2) it immediately follows that the lim $b \to \infty$ of $\partial\theta/\partial\lambda$ equals zero.

5.A.5. Utility maximization

We examine under what circumstances Condition IIa–u is satisfied for some well–known classes of utility flow functions. If $u(x)=x^c$ for $x \geq 0$, with $0<c<\infty$, then Condition IIa–u is equivalent to Condition IIa, so then these are satisfied for the same wage offer distributions. If u is a CARA utility flow function ($u(x)= 1-e^{-cx}$ for $x \geq 0$ with $0<c<\infty$) then Condition IIa–u is weaker than Condition IIa. In fact, it can be shown that the former condition is equivalent to

$$\frac{d \log \psi(x)}{d \log x} \geq - \frac{cx}{1-e^{-cx}} \qquad \text{for every } x \in <\alpha,\beta>$$

For every $c \in <0,\infty>$ and every $x \in <0,\infty>$ the r.h.s. of this inequality is smaller than -1.

The logarithmic function is frequently used to model the utility flow function in a structural empirical search framework (see for example Narendranathan & Nickell (1985), Ridder & Gorter (1986) and van den Berg (1990b)). If $u(\alpha) \geq 0$ (that is, $\alpha \geq 1$) then Condition IIa–u is weaker than Condition IIa. The former condition here equals

$$\frac{d \log \psi(x)}{d \log x} \geq -1 - \frac{1}{\log x} \qquad \text{for every } x \in <\alpha,\beta> \quad (\alpha \geq 1)$$

If $u(\alpha)<0$ $(0 \leq \alpha < 1)$ then Conditions IIa–u and IIa are not nested. However, $\phi>1$ because $u(\phi)>0$, and it can be shown that sufficient for Condition Ia–u to hold is that $\psi(x).u(x)/u'(x)$ is non–decreasing for every $x \in <\phi,\beta>$. Therefore, if $u(x)=\log x$ then Condition Ia–u is weaker than Condition IIa. In sum, for all utility flow functions considered the conditions on the shape of F for $\partial\theta/\partial\lambda$ to be non–negative in search models with utility maximization are weaker than the corresponding conditions in models with income maximization.

We now check whether the second term of the r.h.s. of equation (11) does not decrease in b for some well–known classes of utility flow functions. Using equation (10) one can show that if $u(x)=x^c$ for $x \geq 0$ with $0<c<\infty$, then the second term always increases in b. If u is logarithmic then obviously the second term equals 1. Again it should be noted that for these utility flow functions $|\partial\log\theta/\partial\log b|$ is not decreasing in b under conditions more general than Condition IIa. On the other hand, if u is a CARA utility flow function then the second term (and, in fact, the whole r.h.s. of equation (11)) increases in b for small values of b but not necessarily for large values of b.

5.A.6. Proof of Proposition 4

Let t be an arbitrary point in $[0,T\rangle$. By rewriting equation (14) it follows that $\theta'(t)\leq 0$ if and only if

$$\frac{\lambda'(t)}{\lambda(t)} \leq \psi(\phi(t)).\phi'(t)$$

By substituting equations (12),(A1) and (13) we obtain

$$\frac{\lambda'(t)}{\lambda(t)} \leq \psi(\phi(t)).\rho.(\phi(t)-b) - \psi(\phi(t)).\theta(t).(\mu(\phi(t))-\phi(t))$$

which, because $\mu'(x)=\psi(x).(\mu(x)-x)$ (see Appendix 5.A.1), can be rewritten as

$$\frac{\lambda'(t)}{\lambda(t)} \leq \psi(\phi(t)).\rho.(\phi(t)-b) - \theta(t).\mu'(\phi(t))$$

Because $\phi(t)>b$, sufficient for this to hold is that

$$\frac{\lambda'(t)}{\lambda(t)} \leq - \theta(t).\mu'(\phi(t))$$

A sufficient condition for this inequality can be obtained by noting that $0<\theta(t)<\lambda(t)$,

$$\lambda'(t) \leq - \lambda^2(t).\mu'(\phi(t))$$

If for every $x\in\langle\alpha,\beta\rangle$ inequality (15) is satisfied then clearly the inequality above is also satisfied. Therefore, if for every $x\in\langle\alpha,\beta\rangle$ and for every $t\in[0,T\rangle$ inequality (15) is satisfied, then for every $t\in[0,T\rangle$ $\theta'(t)\leq 0$. Because the left-hand limits of $\lambda'(t)$ and $\phi'(t)$ exist at t=T it follows that the left-hand limit of $\theta'(t)$ at T exists and is non-positive.

Suppose Condition 2a is satisfied. Then for every $x\in\langle\alpha,\beta\rangle$ $\mu'(x)\leq 1$. If in addition $\lambda'(t)\leq-\lambda^2(t)$ (or, equivalently, $-\lambda'(t)/\lambda^2(t)\geq 1$) then inequality (15) is satisfied. Analogously one can show the sufficiency of (17) for (15).

CHAPTER 6

CONCLUSION

The essays in this dissertation deal with microeconometric models of individual labour market behaviour. The emphasis is on structural analysis of transitions between different states on the labour market and durations in those states. Chapter 1 briefly describes the common themes and the distinctive features of the different studies.

In Chapter 2 we specify and estimate a structural job search model for the unemployed that allows for transitions from unemployment into nonparticipation. Moreover, one version of the model deals with the influence of prospective wage increases during employment on the search behaviour of the unemployed. The model is estimated using Dutch data from the mid-eighties. The results indicate that almost every job offer is acceptable. The reason for this is the combination of a very small job offer arrival rate and low values of utility in unemployment relative to employment. If one turns down an offer, then generally one has to wait for a very long time before the next offer arrives. In the meantime one remains unemployed, which is disliked both for pecuniary and for non-pecuniary reasons. The results imply that at an individual level a decrease in benefits is ineffective in reducing unemployment duration. For groups of individuals that almost never get a job offer, about half of the spells of unemployment end in a transition into nonparticipation.

The estimation results in Chapter 2 appear to be robust to varying certain assumptions underlying the empirical model specification. However, throughout the chapter the assumption of stationarity is maintained, which may bias the results. Chapter 3 introduces nonstationarity in job search theory. In a general setting the consequences of nonstationarity for the optimal strategy of unemployed individuals and the duration of unemployment are examined in great detail. We also present comparative dynamics results. Furthermore, by assuming the time-varying explanatory variables to be step functions of time, we are able to derive additional properties of the time path of the optimal strategy. Generally these properties are in accordance with economic intuition.

In the second part of Chapter 3 it is shown that the results derived in the first part can be fruitfully used for the empirical analysis of unemployment durations. As an empirical illustration we estimate a nonstationary structural job search model that allows for the level of benefits to be a decreasing function of unemployment duration, using a Dutch retrospective survey from 1983. For individuals with a low-to-medium level of education, the results indicate that, since the probability of getting a job during the first two years of unemployment is rather small, the anticipation of the decrease of the benefits level that occurs after about two years of unemployment is quite strong. One may say that for the short-term unemployed individuals it holds that the benefits level for the long-term unemployed is an important determinant of their strategy because they know they may well become long-term unemployed themselves. The estimated model can be used for simulating alternative benefits policies like shifting a part of the time path of the level of benefits. Note that all these results cannot be obtained by using stationary models. Moreover, the analysis in Chapter 3 reveals that the reduced-form models of unemployment duration that are generally used to analyze the influence of the benefits level on duration, are not able to represent some of the essential features of nonstationarity due to decreasing benefits. However, it should be noted that the specification of the structural model estimated in Chapter 3 is rather restrictive. Also, in order to be able to estimate the model, extensive use is made of subjective information concerning the optimal strategy and the wage offer distribution. A topic for further research would be to relax some of the rigidities of the model specification and to examine the reliability of the subjective information.

In Chapter 4 we analyse the labour market behaviour of employed individuals by estimating a structural on-the-job search model. This model allows for non-zero and wage-dependent costs associated with moving to another job. It is shown that the optimal strategy of an employed individual has the reservation wage property if the costs of moving do not increase too fast as a function of the wage. The model is estimated using a Dutch retrospective survey from 1985. The results indicate that the state of present housing, age and characteristics of the present job have a large influence on the willingness to move. At an individual level, an increase of the job offer arrival rate has a larger positive effect on job mobility than a decrease of the costs associated with moving to another job.

The empirical analysis in Chapter 4 is flexible in the sense that identification of the structural parameters of interest is achieved without the need to make strong assumptions on certain distributions and other (structural) parameters. It appears that the estimation results satisfy all non-imposed properties of the theoretical model. Moreover, extensive sensitivity analysis of the results shows that these are robust to varying a number of assumptions underlying the empirical model specification. Nevertheless, it seems interesting to extend the model by allowing jobs to have more stochastic characteristics than the wage only. Because, just as in Chapter 3, the data used to estimate the model include subjective responses of individuals on their strategy, another topic for future research is to examine the quality of these responses.

A more general issue for further research would be the evaluation of the predictive power of structural job search models like those used in this thesis. In particular it would be interesting to examine whether a model estimated with data collected prior to a major policy change is able to predict individual behaviour after the policy change. If the policy change occurs just after the collection of the data used to estimate the model, then it may be hard to distinguish between the effects of the anticipation of that change and other determinants of behaviour, like duration dependence of the arrival rate of job offers. Therefore there should preferably be a relatively large time span between the dates at which the data to estimate the model are collected and the date at which the policy change occurs. This implies that panel data are needed following individuals for a large number of years.

The issue of Chapter 5 is the sign of the effect of an increase of the job offer arrival rate on the expected duration of unemployment. It is shown that previously derived sufficient conditions on the wage offer distribution for this effect to be negative in stationary job search models can be weakened at no cost, to include virtually every conceivable empirical wage offer distribution. In particular, all families of distributions generally used to model wage offer distributions in structural job search models and other income-related distributions do not satisfy the conditions derived before but do satisfy the conditions presented in Chapter 5.

The results in Chapter 5 have some implications for both structural and reduced-form empirical analyses of unemployment duration. First of all, the interpretation of the estimates of reduced-form models is facilitated. For instance, if an explanatory variable is found to be positively related to duration, and this variable is believed not to influence determinants of duration other than the job offer arrival rate, then it can be concluded that the variable has a negative influence on the job offer arrival rate. Furthermore, since it is likely that small changes of the shape of the wage offer distribution do not imply a sign-reversal of the relationship between the job offer arrival rate and duration, one may say that in this sense the estimates of structural models are robust with respect to the assumed family of wage offer distributions.

By now the use of job search theory for the theoretical and empirical analysis of individual labour market behaviour over time has become widespread. Structural empirical analysis based on this theory seems to provide a fruitful platform for a deepening of economic insights into this behaviour. This thesis provided some new plays on this platform.

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SAMENVATTING

Dit proefschrift bevat micro-econometrische analyses van het gedrag van individuen op de arbeidsmarkt. Uitgangspunt is de z.g. 'job search theory', een micro-economische theorie die dit gedrag poogt te verklaren, met name voor zover dat betrekking heeft op het zoeken naar een nieuwe of andere baan. Deze theorie houdt expliciet rekening met verschillende vormen van onzekerheid zoals die zich voordoen op de arbeidsmarkt. Verondersteld wordt dat beslissingen van individuen geleid worden door het streven het verwachte nut te maximaliseren. Op basis van de theorie worden modellen geconstrueerd die geschat worden met behulp van econometrische technieken. Hierbij wordt gebruik gemaakt van gegevens uit omvangrijke enquêtes onder werkenden en werklozen in Nederland.

De gebruikte modellen zijn struktureel van aard. Dit betekent dat het theoretische kader van de 'job search theory' de algemene vorm van de modellen vastlegt. In het bijzonder maken de vergelijkingen die de strategie beschrijven die (volgens de theorie) ten grondslag ligt aan het gedrag van individuen, deel uit van de te schatten modellen. Het schatten van zulke modellen maakt het mogelijk gedetailleerde uitspraken te doen over het arbeidsmarktgedrag van werklozen en werkenden en het hoe en waarom van de (in-)effektiviteit van bepaalde beleidsmaatregelen ter vermindering van de werkloosheidsduren en vergroting van de arbeidsmobiliteit. De schattingsmethoden die aan de orde komen zijn de z.g. 'maximum likelihood' methode en de niet-lineaire kleinste-kwadraten methode.

Hoofdstuk 1 bevat een korte beschrijving van de gemeenschappelijke thema's en onderscheidende kenmerken van de daaropvolgende hoofdstukken.

In Hoofdstuk 2 wordt een model voor werklozen geconstrueerd en geschat dat toelaat dat men als werkloze uit de arbeidsmarkt stapt. Een uitgebreide versie van het model houdt bovendien rekening met de invloed op het gedrag van werklozen van verwachte loonstijgingen in een eventuele toekomstige baan. Beide versies worden geschat met gebruik van gegevens uit het midden van de jaren tachtig. Uit de resultaten volgt dat voor werklozen bijna iedere aangeboden baan acceptabel is. Een verlaging van het uitkeringsniveau is ineffektief in het verlagen van de werkloosheidsduur. In groepen individuen die praktisch nooit een baan aangeboden krijgen stapt ongeveer de helft van de werklozen uit de arbeidsmarkt voordat een baan is gevonden.

Hoofdstuk 3 introduceert niet-stationariteit in 'job search theory'. In een algemeen kader wordt onderzocht wat de gevolgen voor de strategie van werklozen en voor de werkloosheidsduur zijn als wordt toegelaten dat verklarende variabelen als het uitkeringsniveau en de verdeling van aangeboden lonen variëren met de werkloosheidsduur. Aangetoond wordt dat de evolutie van de optimale strategie van werklozen over de tijd kan worden beschreven met een differentiaalvergelijking. Hoe specifieker de veronderstellingen die gemaakt worden over de manier waarop de verklarende variabelen met de werkloosheidsduur variëren, hoe gedetailleerder de eigenschappen die afgeleid worden voor de oplossing van die vergelijking.

In het tweede deel van Hoofdstuk 3 wordt aangetoond dat de resultaten uit het eerste deel op een vruchtbare manier kunnen worden gebruikt voor de empirische analyse van werkloosheidsduren. Als illustratie wordt een model geschat dat rekening houdt met het feit dat het niveau van de werkloosheidsuitkering in het algemeen daalt als de werkloosheidsduur een bepaalde periode overschrijdt. Hiervoor worden retrospectieve gegevens uit 1983 gebruikt. Het blijkt dat individuen met een laag of gemiddeld opleidingsniveau zulke dalingen van het uitkeringsniveau sterk anticiperen.

Hoofdstuk 4 bevat een analyse van het arbeidsmarktgedrag van werkende individuen. Het is aannemelijk dat de kosten die verbonden zijn aan het veranderen van baan behoren tot de belangrijkste faktoren die inflexibiliteit van de arbeidsmarkt veroorzaken. Daarom wordt bij de analyse speciale aandacht aan deze kosten gegeven. Aangetoond wordt dat de optimale strategie van een werkend individu de reserveringsloon-eigenschap heeft als de kosten verbonden aan het veranderen van baan niet te sterk stijgen als een funktie van het loon. Het model wordt geschat met gebruik van retrospectieve gegevens uit 1985. Het blijkt dat de situatie op de woningmarkt, leeftijd en karakteristieken van de huidige baan een sterke invloed hebben op de mate van bereidheid om van baan te veranderen, terwijl de gezinssituatie en de gehechtheid aan de omgeving hier een relatief geringe rol spelen. Op een individueel niveau heeft een relatieve vergroting van het verwachte aantal aangeboden banen in een bepaalde periode een groter positief effekt op de arbeidsmobiliteit dan een relatieve verlaging van de kosten verbonden aan het veranderen van baan.

Hoofdstuk 5 is theoretisch van aard en bevat geen empirische resultaten. Het onderwerp van dit hoofdstuk is de relatie in 'job search theory' tussen, enerzijds, het verwachte aantal banen dat een werkloos individu in een bepaalde periode aangeboden krijgt (oftewel de snelheid waarmee banen worden aangeboden) en, anderzijds, de verwachte werkoosheidsduur. In het algemeen wordt onderkend dat een vergroting van de snelheid waarmee banen worden aangeboden twee tegengestelde effekten heeft op de verwachte werkloosheidsduur: een negatief effekt vanwege het gemiddeld groter aantal mogelijkheden om de toestand van werkloosheid te verlaten, en een positief effekt vanwege een kritischer houding van de werkloze ten opzichte van banen juist als gevolg van de gemiddeld grotere keuzemogelijkheden. In Hoofdstuk 5 wordt aangetoond dat elders in de literatuur afgeleide voorwaarden op de verdeling van aangeboden lonen die er voor zorgen dat het negatieve effekt domineert, enorm afgezwakt kunnen worden. Alle families van kansverdelingen die in het algemeen gebruikt worden om de verdeling van aangeboden lonen en inkomensverdelingen te modelleren voldoen niet aan de elders afgeleide voorwaarden maar wel aan de voorwaarden die in Hoofdstuk 5 worden afgeleid.

De resultaten in Hoofdstuk 5 hebben implicaties voor de empirische analyse van werkloosheidsduren. Zo wordt de interpretatie van de schattingen van gereduceerde-vorm modellen vergemakkelijkt. Bovendien zijn schattingen van strukturele modellen robuust ten aanzien van de specificatie van de verdeling van aangeboden lonen, in die zin dat het aannemelijk is dat kleine veranderingen in de specificatie van die verdeling geen tekenverandering impliceren in de relatie tussen de snelheid waarmee banen worden aangeboden en de werkloosheidsduur.

Hoofdstuk 6 bevat een korte samenvatting en evaluatie van de verschillende resultaten.